

# The binomial expansion

72 marks



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Date \_\_\_\_\_

## **A** Do Now

Quick questions to start.

- |                                                                                    |                                                                                      |
|------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| <b>A1</b> [Last lesson] [2 marks]<br>Work out $\binom{8}{3}$ without a calculator. | <b>A2</b> [Last lesson] [2 marks]<br>Find the coefficient of $x^2$ in $(1 + 3x)^4$ . |
|------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|

Answer .....

Answer .....

- |                                                                                               |                                                                                     |
|-----------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| <b>A3</b> [From chapter 1] [2 marks]<br>Simplify $\left(\frac{3}{x}\right)^2 \times (2x)^3$ . | <b>A4</b> [From chapter 2] [2 marks]<br>Solve $4k^2 = 9k$ , given that $k \neq 0$ . |
|-----------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|

Answer .....

Answer .....

Your teacher works through each example with you. Then try the one beside it.

**EXAMPLE 1**

Find the first four terms, in ascending powers of  $x$ , of  $(1 - 3x)^9$ .

**Hint**

$$(1 + b)^n = 1 + nb + \binom{n}{2}b^2 + \binom{n}{3}b^3 + \dots$$

with  $b = -3x$ .

**B1** Find the first three terms, in ascending powers of  $x$ , of  $(2 + \frac{1}{4}x)^{10}$ . [3 marks]

Answer .....

**EXAMPLE 2**

Find the coefficient of  $x^5$  in the expansion of  $(2 - x)^{11}$ .

**Hint** The general term is  $\binom{n}{r}a^{n-r}b^r$ . Here  $r = 5$ ,  $a = 2$ ,  $b = -x$ .

**B2** Find the coefficient of  $x^6$  in the expansion of  $(1 - 2x)^{10}$ . [2 marks]

Answer .....

**EXAMPLE 3**

Find the term independent of  $x$  in the expansion of  $(x + \frac{2}{x})^6$ .

**Hint** The general term holds  $x^{6-r}(\frac{2}{x})^r$ , which is  $x^{6-2r}$ . Make the power 0.

**B3** Find the term independent of  $x$  in the expansion of  $(2x^2 - \frac{1}{x})^9$ . [3 marks]

Answer .....

**EXAMPLE 4**

In the expansion of  $(1 + kx)^n$ , the coefficient of  $x$  is 15 and the coefficient of  $x^2$  is 90. Find  $n$  and  $k$ .

**Hint**  $nk = 15$  and  $\frac{n(n-1)}{2}k^2 = 90$ . Put  $k = \frac{15}{n}$  into the second.

**B4** In the expansion of  $(1 + kx)^n$ , the coefficient of  $x$  is  $-24$  and the coefficient of  $x^2$  is 252. Find  $n$  and  $k$ . [4 marks]

Answer .....

**c Practice**

A-Level

**C1** Find the binomial expansion of  $\left(2x - \frac{1}{x}\right)^5$ , giving each term in its simplest form. [4 marks]

Answer .....

- C2** In the expansion of  $(1 + 2x)^n$ , where  $n \geq 3$ , the coefficient of  $x^3$  is 4 times the coefficient of  $x^2$ . Find  $n$ . [4 marks]

Answer .....

- C3** Find the coefficient of  $x^3$  in the expansion of  $(1 - x)^2(1 + 3x)^6$ . [4 marks]

Answer .....

## **D** Exam-style questions

Edexcel exam style

- D1** Ali writes: "The  $x^3$  term of  $\left(2 - \frac{x}{2}\right)^8$  is  $\binom{8}{3} 2^5 \left(\frac{x}{2}\right)$ , so the coefficient of  $x^3$  is  $56 \times 32 \times \frac{1}{2} = 896$ ."

Ali's answer is wrong.

- (a)** Describe the two mistakes in Ali's working. [2 marks]

(b) Find the correct coefficient of  $x^3$ .

[2 marks]

Answer .....

(c) Hence find the coefficient of  $x^3$  in the expansion of  
 $(1 + 4x) \left(2 - \frac{x}{2}\right)^8$ .

[3 marks]

Answer .....

Total for D1: 7 marks

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**D2**  $p$  is a positive constant.

(a) Find the first three terms, in ascending powers of  $x$ , of the binomial expansion of  $(1 + px)^{12}$ .

[2 marks]

Answer .....

(b) In this expansion, the coefficient of  $x^2$  is 22 times the coefficient of  $x$ . Find  $p$ .

[3 marks]

Answer .....

- (c) Hence find the coefficient of  $x^3$  in the expansion of  $(3 - x)(1 + px)^{12}$ .

[4 marks]

Answer .....

Total for D2: 9 marks

- D3**  $a$  is a constant. In the expansion of  $\left(ax - \frac{3}{x^2}\right)^6$ , the term independent of  $x$  is 2160.

- (a) Find the two possible values of  $a$ .

[4 marks]

Answer .....

- (b) Hence, for  $a > 0$ , find the coefficient of  $x^3$  in this expansion.

[3 marks]

Answer .....

(c) Explain why there is no term in  $x^2$  in this expansion.

[2 marks]

Total for D3: 9 marks

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**D4** In this question, do not use a calculator.

(a) Show that  $(x + y)^5 - (x - y)^5 = 10x^4y + 20x^2y^3 + 2y^5$ .

[3 marks]

(b) Hence show that  $(2 + \sqrt{3})^5 - (2 - \sqrt{3})^5 = k\sqrt{3}$ , and find the integer  $k$ .

[3 marks]

Answer .....

(c) Find the exact value of  $(2 + \sqrt{3})^5 + (2 - \sqrt{3})^5$ .

[4 marks]

Answer .....

Total for D4: 10 marks

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No route is given. The hints are at the end of the sheet. Use one at a time.

**E1** Find the coefficient of  $x^3$  in the expansion of  $(1 + x + x^2)^{10}$ . [5 marks]

Answer .....

## Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write  $1 + x + x^2$  as  $1 + y$ , with  $y = x + x^2$ .  
**Hint 2** Only the  $y^2$  and  $y^3$  terms of  $(1 + y)^{10}$  give  $x^3$ .  
**Hint 3**  $y^2$  holds  $2x^3$  and  $y^3$  holds  $x^3$ .

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write the first terms of a binomial expansion in ascending powers of  $x$ . ☐ ☐ ☐

I can find one coefficient from the general term. ☐ ☐ ☐

I can find a term independent of  $x$  when both terms hold  $x$ . ☐ ☐ ☐

I can find an unknown constant or  $n$  from given coefficients. ☐ ☐ ☐