The chain rule

114 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-the-chain-rule/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Write $\sqrt{3x-1}$ and $\dfrac{4}{(2x+5)^3}$ in the form $a(px+q)^n$.

Answer .....

A2 Last lesson [3 marks]

Differentiate $y = 6e^{4x} - 2\ln(3x) + 5\sin 2x$.

Answer .....

A3 Common slip [2 marks]

Sam says the derivative of $(2x+1)^5$ is $5(2x+1)^4$. What has he left out? Give the correct derivative.

Answer .....

A4 Needed today [2 marks]

Given $x = y^3 + 2y$, find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ and its value when $y = 1$.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Given $y = (3x^2 - 5)^4$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

Hint Let u be the bracket. Then y = u to the 4. Multiply the derivative of u to the 4 by the derivative of u.

B1 Given $y = (2 - 5x^3)^6$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and the gradient of the curve at the point where $x = 1$. [4 marks]

Answer .....

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EXAMPLE 2

Find the gradient of the curve $y = \sqrt{4x^2 + 9}$ at the point where $x = 2$.

Hint Write the root as a power of one half. Bring the half down, take one from the power, and multiply by the derivative of the inside.

B2 [5 marks]

The curve C has equation $y = \dfrac{6}{(x^2 + 3)^2}$. Find an equation of the tangent to C at the point where $x = 1$, in the form $y = mx + c$.

Answer .....

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EXAMPLE 3

Differentiate (a) $y = e^{x^2-3x}$ (b) $y = \ln(\cos x)$.

Hint For e to a function, multiply by the derivative of the power. For ln of a function, divide the derivative of the inside by the inside.

B3 Differentiate (a) $y = \ln(5 - x^2)$ (b) $y = \cos^4(2x)$. [5 marks]

Answer .....

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EXAMPLE 4

The curve C has equation $x = y^2 - 4y$. Find the gradient of C at the point $(-3, 1)$.

Hint Differentiate x with respect to y, then turn it upside down.

B4 The curve C has equation $x = 2y^3 + y - 1$. Find an equation of the tangent to C at the point where $y = 1$, in the form $y = mx + c$. [5 marks]

Answer .....

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EXAMPLE 5

Given $x = e^{2y}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

Hint Find dx/dy, write it in terms of x, then take 1 over it.

B5 Given $x = (y+2)^3$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x. [4 marks]

Answer .....

C Practice

A-Level

C1 Kai writes $\dfrac{\mathrm{d}}{\mathrm{d}x}(4x-3)^{-2} = -8(4x-3)^{-1}$ and $\dfrac{\mathrm{d}}{\mathrm{d}x}\sqrt{x^2+1} = \dfrac{1}{2\sqrt{x^2+1}}$. Find his mistake in each and give the correct derivatives. [4 marks]

Answer .....

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C2 Find the coordinates of the stationary points on the curve $y = (x^2 - 4)^3$ and decide the nature of each. [6 marks]

Answer .....

C3 The curve $x = y^2 + 2y$ passes through two points with $x = 3$. Find the gradient of the curve at each of them. [4 marks]

Answer .....

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C4 Show that the curve $y = \ln(x^2 + 2x + 5)$ has exactly one stationary point. Find its exact coordinates and its nature. [5 marks]

Answer .....

C5 Lena says that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(e^{x^2}\right) = x^2 e^{x^2 - 1}$. Explain why she is wrong, and find the exact gradient of $y = e^{x^2}$ at $x = 1$. [4 marks]

Answer .....

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D1 The curve C has equation $y = \dfrac{4}{x^2 - 6x + 13}$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{8(3 - x)}{(x^2 - 6x + 13)^2}$. [3 marks]

(b) Find the coordinates of the stationary point of C and decide its nature. [4 marks]

Answer .....

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(c) Find an equation of the normal to C at the point where $x = 1$, in the form $y = mx + c$. [4 marks]

Answer .....

Total for D1: 11 marks

D2

> Omar wants the tangent to the curve $x = y^3 - 3y$ at the point $(2, 2)$. He writes: "$\frac{\mathrm{d}y}{\mathrm{d}x} = 3y^2 - 3 = 9$ at the point, so the tangent is $y - 2 = 9(x - 2)$."

Omar is wrong.

(a) Explain his mistake. [2 marks]

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(b) Find the correct equation of the tangent in the form $y = mx + c$. [3 marks]

Answer .....

(c) Find the coordinates of the points on the curve where the tangent is parallel to the y-axis. [4 marks]

Answer .....

Total for D2: 9 marks

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D3 Show that the gradient of the curve $y = \ln(x^2 + 1)$ is never greater than 1. Find the point where the gradient equals 1, and an equation of the tangent at that point. [9 marks]

Answer .....

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D4 The number of birds, P, on an island t years after they arrive is modelled by $P = \dfrac{800}{1 + 3e^{-0.4t}}$, $t \geq 0$.

(a) Find the number of birds when they arrive. [2 marks]

Answer .....

(b) Show that $\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{960e^{-0.4t}}{(1 + 3e^{-0.4t})^2}$. [3 marks]

(c) Find the rate at which the number of birds is growing when $t = 5$. [2 marks]

Answer .....

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(d) Find the exact time at which there are 400 birds, and give it to 3 significant figures. [3 marks]

Answer .....

(e) State the number of birds the model gives in the long term, and give one reason why the model may not be right for large values of t. [2 marks]

Total for D4: 12 marks

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D5 The curve C has equation $x = e^{2y} - 4y$.

(a) Find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$. [2 marks]

Answer .....

(b) Find the exact coordinates of the point on C where the tangent is parallel to the y-axis. [3 marks]

Answer .....

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(c) Hence, or otherwise, find an equation of the normal to C at the point where $y = 0$. [4 marks]

Answer .....

Total for D5: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The curve C has equation $y = \ln(x^2 + ax + 9)$, where a is a constant. Find the set of values of a for which C has a stationary point. Find the values of a for which this stationary point has y-coordinate $\ln 5$. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Differentiate with the chain rule. Where can the top of your fraction be zero?

Hint 2 A logarithm only exists when the inside is positive. Is the curve defined at the x you found?

Hint 3 Put $x = -\frac{a}{2}$ into $x^2 + ax + 9$. When is this positive, and when does it equal 5?

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can differentiate a function of a function, such as a power of a bracket, e to a function or ln of a function.	○ ○ ○
I can use the chain rule more than once when one function sits inside another inside another.	○ ○ ○
I can use the fact that the gradient is 1 divided by the rate of change of x with respect to y.	○ ○ ○
I can use these to find tangents, normals and stationary points.	○ ○ ○

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