

The cosine rule and the sine rule

70 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-the-cosine-rule-and-the-sine-rule/

Date _____

A Do Now

Quick questions to start.

- | | |
|---|--|
| <p>A1 Needed today [2 marks]</p> <p>A right-angled triangle has hypotenuse 13 cm. Find the side opposite an angle of 35°, to 3 s.f.</p> | <p>A2 Needed today [2 marks]</p> <p>Solve $x^2 + 4x - 17 = 0$, giving your answers in the form $a \pm \sqrt{b}$.</p> |
|---|--|

Answer

Answer $x = \dots\dots$

- | | |
|---|---|
| <p>A3 Last lesson [2 marks]</p> <p>Estimate 1.02^{10} using the first three terms of the expansion of $(1 + x)^{10}$.</p> | <p>A4 From chapter 6 [2 marks]</p> <p>Find the centre and the radius of the circle $x^2 + y^2 - 6x + 8y = 11$.</p> |
|---|---|

Answer

Answer

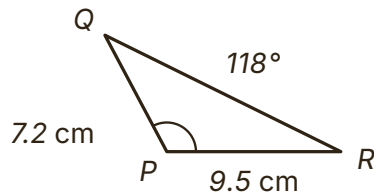
B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

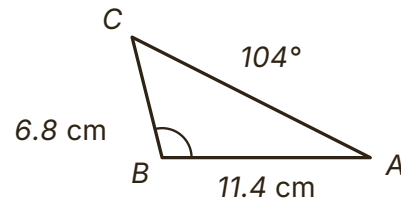
EXAMPLE 1

Triangle PQR is shown. Find QR , to 3 s.f.



Hint You know two sides and the angle between them, so use the cosine rule. $\cos 118^\circ$ is negative, so the last term adds on.

B1 Triangle ABC is shown. [3 marks]
Find AC , to 3 s.f.

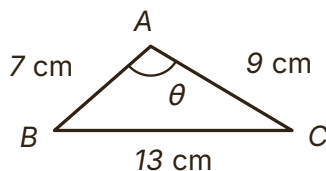


Not drawn accurately

Answer

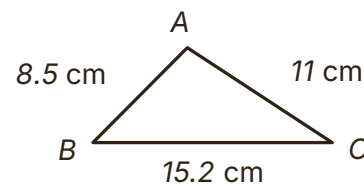
EXAMPLE 2

Find the size of the largest angle of this triangle.



Hint The largest angle faces the longest side. Rearrange the cosine rule for $\cos \theta$.

B2 Find the size of the smallest angle of this triangle. [3 marks]

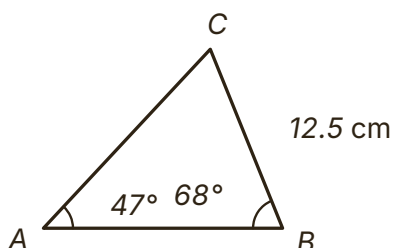


Not drawn accurately

Answer

EXAMPLE 3

Triangle ABC is shown. Find AB , to 3 s.f.

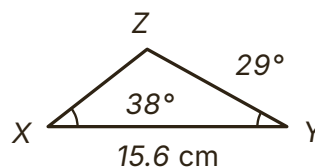


Hint Find angle C first. Pair AB with $\sin C$ and BC with $\sin A$.

B3

Triangle XYZ is shown. Find YZ , to 3 s.f.

[3 marks]

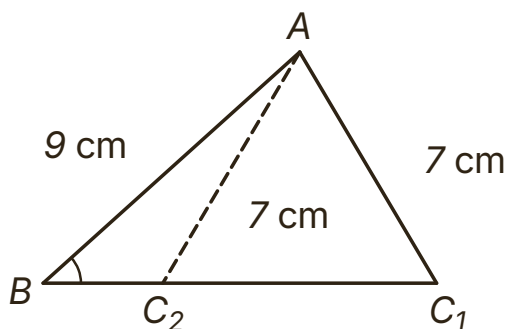


Not drawn accurately

Answer

EXAMPLE 4

In triangle ABC , $AB = 9\text{ cm}$, $AC = 7\text{ cm}$ and angle $ABC = 42^\circ$. Find the two possible values of angle ACB .



Hint $\sin x = \sin(180^\circ - x)$; try the acute and the obtuse angle. Check each fits with the 42° .

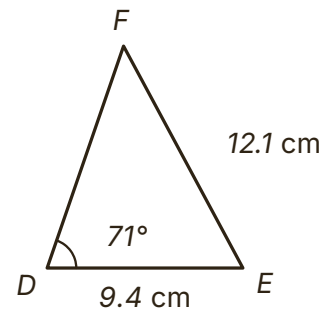
B4

In triangle DEF , $DE = 14\text{ cm}$, $DF = 11\text{ cm}$ and angle $DEF = 41^\circ$. Find the two possible values of angle DFE , and for each the size of angle EDF .

[4 marks]

Answer

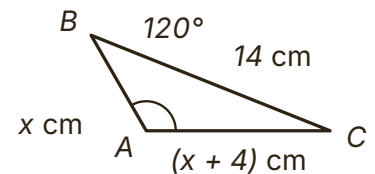
- C1** In triangle DEF , $DE = 9.4$ cm, $EF = 12.1$ cm and angle $EDF = 71^\circ$. Find angle DFE , and explain why it has only one possible value. [4 marks]



Not drawn accurately

Answer angle $DFE = \dots\dots$

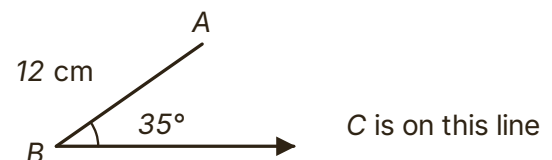
- C2** Triangle ABC is shown, with lengths in centimetres. Find the value of x . [4 marks]



Not drawn accurately

Answer $x = \dots\dots$

- C3** Triangle ABC has $AB = 12$ cm and angle $ABC = 35^\circ$. Find the range of values of the length AC for which two different triangles ABC can be drawn. [4 marks]



Answer $\dots\dots < AC < \dots\dots$

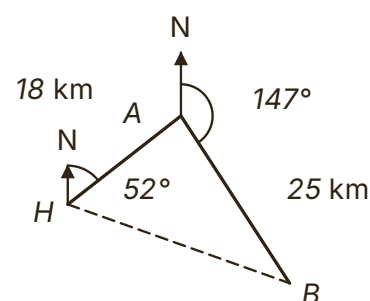
- C4** In triangle ABC the sides a , b and c satisfy $a^2 = b^2 + c^2 + bc$. Show that angle $A = 120^\circ$. [3 marks]

D Exam-style questions

Edexcel exam style

- D1** A boat leaves harbour H and sails 18 km on a bearing of 052° to a buoy A . It then sails 25 km on a bearing of 147° to a buoy B .

- (a) Show that angle $HAB = 85^\circ$. [2 marks]



- (b) Hence find the distance HB , to 3 s.f. [3 marks] Not drawn accurately

Answer

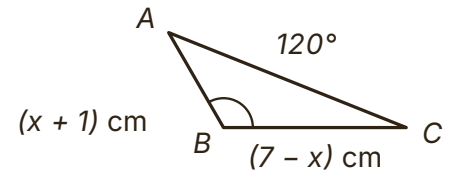
- (c) Find the bearing of H from B , to the nearest degree. [3 marks]

Answer $^\circ$

Total for D1: 8 marks

D2 In triangle ABC , $AB = (x + 1)$ cm, $BC = (7 - x)$ cm and angle $ABC = 120^\circ$, where $-1 < x < 7$.

(a) Show that $AC^2 = x^2 - 6x + 57$. [3 marks]



Not drawn accurately

(b) Hence find the exact minimum length of AC , and the value of x at which it happens. [3 marks]

Answer $AC = \dots\dots$ $x = \dots\dots$

Total for D2: 6 marks

D3 In triangle PQR , $PQ = 11$ cm, $QR = 8$ cm and angle $QPR = 40^\circ$. Kai writes: $\sin R = \frac{11 \sin 40^\circ}{8} = 0.884$, so angle $PRQ = 62.1^\circ$.

Kai says this is the only possible value. Kai is wrong.

(a) Explain the mistake Kai has made. [2 marks]

(b) Find the size of angle PQR in each of the two triangles. [2 marks]

Answer $\dots\dots$ or $\dots\dots$

(c) Hence find the two possible lengths of PR , to 3 s.f.

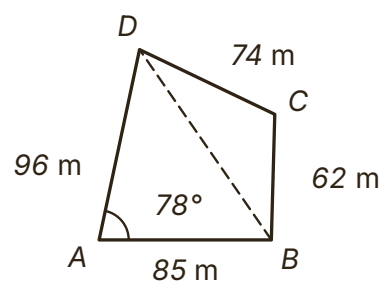
[4 marks]

Answer cm or cm

Total for D3: 8 marks

D4 The diagram shows a field $ABCD$. $AB = 85$ m, $BC = 62$ m, $CD = 74$ m, $DA = 96$ m and angle $BAD = 78^\circ$.

- (a) Find the length of the diagonal BD , [3 marks]
to 3 s.f.



Not drawn accurately

Answer

- (b) Hence find the size of angle BCD .

[3 marks]

Answer

Total for D4: 6 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The sides of a triangle are three consecutive whole numbers of centimetres, and its largest angle is twice its smallest angle. Find the lengths of the three sides. [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Bisect the largest angle. Look for an isosceles triangle and a pair of similar triangles.
- Hint 2** Smallest angle θ at A (side $a = n - 1$), angle 2θ at B (side $b = n + 1$), $c = n$. The bisector of angle B meets AC at D : $AD = BD$, and triangle BDC is similar to triangle ABC .
- Hint 3** Similar triangles: $DC = \frac{a^2}{b}$, $AD = BD = \frac{ac}{b}$. $AD + DC = b$ gives $b^2 = a(a + c)$, so $(n + 1)^2 = (n - 1)(2n - 1)$, $n = 5$: sides 4, 5, 6.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use the cosine rule to find a side or an angle. ☐ ☐ ☐

I can use the sine rule to find a side or an angle. ☐ ☐ ☐

I can find both angles when the sine rule gives two. ☐ ☐ ☐

I can solve problems with bearings and algebraic sides. ☐ ☐ ☐