

Midpoints, perpendicular bisectors and the equation of a circle

74 marks



Hints online, answers free
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mathswariz.uk/worksheets/a-level-the-equation-of-a-circle/

Date _____

A Do Now

Quick questions to start.

- | | |
|---|--|
| <p>A1 GCSE Higher [2 marks]</p> <p>Find the midpoint and the length of the line segment joining $(-3, -6)$ and $(5, 0)$.</p> | <p>A2 From chapter 5 [2 marks]</p> <p>Find the equation of the line through $(1, 4)$ perpendicular to $y = 2x - 3$, in the form $ax + by + c = 0$.</p> |
|---|--|

Answer $(\dots, \dots)$

Answer $\dots\dots = 0$

- | | |
|---|--|
| <p>A3 Needed today [2 marks]</p> <p>Write $x^2 - 10x + 7$ in the form $(x - p)^2 + q$.</p> | <p>A4 From chapter 2 [2 marks]</p> <p>Solve $(k - 3)^2 + 16 = 25$.</p> |
|---|--|

Answer

Answer $k = \dots\dots$

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A is $(-3, 2)$ and B is $(7, 6)$. Find the perpendicular bisector of AB , in the form $ax + by + c = 0$.

Hint It passes through the midpoint of AB and is perpendicular to AB .

B1 P is $(-9, -5)$ and Q is $(-1, -1)$. Find the perpendicular bisector of PQ , in the form $ax + by + c = 0$. [4 marks]

Answer = 0

EXAMPLE 2

A circle has centre $(-2, 4)$ and passes through $(4, -4)$. Find its equation, and decide whether $(5, -1)$ is inside, on or outside it.

Hint The radius is the distance from the centre to the point.

B2 A circle has centre $(3, -1)$ and passes through $(8, 11)$. Find its equation, show that $(15, 4)$ lies on it, and decide whether $(-6, 3)$ is inside, on or outside it. [4 marks]

Answer

EXAMPLE 3

Find the centre and the radius of the circle $x^2 + y^2 + 6x - 10y - 2 = 0$.

Hint Complete the square in x and in y separately.

B3 Find the centre and the [4 marks]

radius of the circle

$$2x^2 + 2y^2 - 8x + 12y - 6 = 0.$$

Answer $(\dots, \dots)$, $r = \dots$

EXAMPLE 4

$x^2 + y^2 + 8x - 2y + k = 0$ is the equation of a circle. Find the range of possible values of k , and the value of k when the radius is 3.

Hint After completing the square, the right-hand side is r^2 , and it must be positive.

B4 A circle has equation [4 marks]

$$x^2 + y^2 - 10x - 6y + k = 0.$$

Find the range of possible values of k , and the value of k for which the circle touches the x -axis.

Answer $\dots$ $k = \dots$

- C1** The midpoint of PQ is $(2, -5)$, and P is $(7, 1)$. Find the coordinates of Q , and the exact length of PQ . [3 marks]

Answer $(\dots, \dots)$

- C2** $A(-6, 1)$ and $B(4, -3)$ are the ends of a diameter of a circle. Find the equation of the circle, and show that $(-3, 4)$ lies on it. [4 marks]

Answer

- C3** The point $(k, 5)$ lies on the circle $(x - 2)^2 + (y + 1)^2 = 61$. Find the two possible values of k . [3 marks]

Answer $k = \dots\dots$

- C4** A is $(-1, 3)$ and B is $(5, 7)$. Show that the perpendicular bisector of AB passes through $C(8, -4)$. [4 marks]

D Exam-style questions

Edexcel exam style

- D1** The circle C has equation $x^2 + y^2 + 10x - 4y - 7 = 0$. Ben says: "Halve the 10 and the -4 : the centre of C is $(5, -2)$."

Ben is wrong.

- (a)** Find the centre and the radius of C , and explain Ben's mistake. [5 marks]

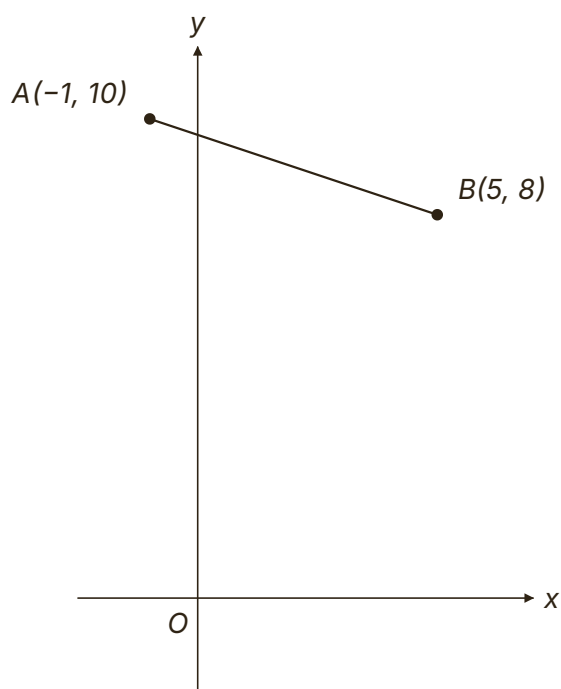
- (b)** Show that $A(1, 2)$ lies on C . Hence find the coordinates of B , where AB is a diameter of C . [3 marks]

Answer $B(\dots, \dots)$

Total for D1: 8 marks

D2 The points $A(-1, 10)$ and $B(5, 8)$ are shown.

- (a) Find the perpendicular bisector of AB , in the form $ax + by + c = 0$. [4 marks]



Not drawn accurately

Answer = 0

- (b) A circle passes through A and B , and its centre lies on the y -axis. Find its equation. [3 marks]

Answer

- (c) Hence find the other point on the circle with y -coordinate 10. [2 marks]

Answer (...,...)

Total for D2: 9 marks

D3 The circle C has equation $x^2 + y^2 - 2kx + 6y + k = 0$, where k is a constant.

- (a) Find the centre of C , and show that its radius r satisfies [3 marks]
 $r^2 = k^2 - k + 9$.

- (b) Hence show that C is a circle for every real value of k . [2 marks]

- (c) Given that the radius of C is $\sqrt{11}$, find the two possible values of k . [3 marks]

Answer $k = \dots\dots$

Total for D3: 8 marks

- D4** Circle C_1 has equation $x^2 + y^2 - 4x - 2y - 4 = 0$ and circle C_2 has equation $x^2 + y^2 + 8x + 14y + 16 = 0$. Find the centre and radius of each, and hence show that they touch. [5 marks]

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A circle touches both axes and passes through $(1, 8)$. Find its two possible equations. [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** It touches both axes, so its centre is (r, r) .
Hint 2 The distance from (r, r) to $(1, 8)$ is r .
Hint 3 $r^2 - 18r + 65 = 0$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- | | |
|--------------------------------|---|
| I can bisect a line segment. | ☐ ☐ ☐ |
| I can write a circle equation. | ☐ ☐ ☐ |
| I can find centre and radius. | ☐ ☐ ☐ |
| I can find a missing value. | ☐ ☐ ☐ |