



The factor theorem

71 marks

Date _____

A Do Now

Quick questions to start.

- | | |
|---|--|
| A1 Needed today [2 marks]
Work out $f(-2)$ when
$f(x) = 2x^3 - 5x^2 + 4x + 7$. | A2 Last lesson [2 marks]
Divide $x^3 + 2x^2 - 7x - 2$
by $(x - 2)$. |
|---|--|

Answer

Answer

- | | |
|--|---|
| A3 Needed today [2 marks]
Factorise $12x^2 + x - 6$. | A4 From chapter 2 [2 marks]
Find the discriminant of
$2x^2 + 3x + 7$ and the
number of real roots. |
|--|---|

Answer

Answer

Work through each example, then try the one beside it.

EXAMPLE 1

Show that $(x - 3)$ is a factor of $f(x) = x^3 - x^2 - 14x + 24$. Hence factorise $f(x)$ completely.

Hint $(x - 3)$ is a factor when $f(3) = 0$. Then divide by $(x - 3)$ and factorise the quadratic.

B1 Show that $(2x - 1)$ is a factor of $f(x) = 2x^3 + 5x^2 - 23x + 10$. Hence factorise $f(x)$ completely. [3 marks]

Answer

EXAMPLE 2

Factorise $x^3 + 6x^2 - x - 30$ completely. Hence solve $x^3 + 6x^2 - x - 30 = 0$.

Hint Try $x = \pm 1, \pm 2, \pm 3, \dots$: a root of $x^3 + \dots - 30$ must divide 30.

B2 Factorise completely $f(x) = 3x^3 - 7x^2 - 7x + 3$. Hence sketch $y = f(x)$, showing where it meets the axes. [5 marks]

EXAMPLE 3

$(x - 2)$ is a factor of
 $f(x) = 2x^3 + ax^2 - 7x - 6$. Find a .
Hence factorise $f(x)$ completely.

Hint $(x - 2)$ is a factor, so $f(2) = 0$. That gives an equation in a .

B3 $(x + 3)$ is a factor of [4 marks]
 $f(x) = 3x^3 + kx^2 - 38x - 24$.
Find k . Hence factorise
 $f(x)$ completely.

Answer $k = \dots\dots ; \dots\dots$

c Practice

A-Level

C1 $g(x) = x^4 + 2x^3 - 13x^2 - 14x + 24$. Show that $(x - 1)$ and $(x + 2)$ [5 marks]
are both factors of $g(x)$. Hence factorise $g(x)$ completely.

Answer

- C2** Given that $(3x + 2)$ is a factor of $6x^3 + 7x^2 + kx - 6$, find the value of k . Hence solve $6x^3 + 7x^2 + kx - 6 = 0$. [5 marks]

Answer $k = \dots$; $x = \dots\dots$

- C3** Given that $(x + 1)$ and $(x - 3)$ are factors of $px^3 + qx^2 - 5x + 6$, find the values of p and q . Hence find the third linear factor. [5 marks]

Answer $p = \dots$, $q = \dots$; $\dots\dots$

D Exam-style questions

Edexcel exam style

- D1** For $f(x) = x^3 + 2x^2 - 13x + 10$, Sam writes: " $f(2) = 8 + 8 - 26 + 10 = 0$, so $(x + 2)$ is a factor of $f(x)$."

Sam is wrong.

- (a)** Explain the mistake Sam has made.

[1 mark]

(b) Factorise $f(x)$ completely.

[3 marks]

Answer

(c) Hence solve $(y - 3)^3 + 2(y - 3)^2 - 13(y - 3) + 10 = 0$.

[2 marks]

Answer $y = \dots\dots$

Total for D1: 6 marks

D2 $f(x) = 3x^3 + ax^2 + bx + 4$, where a and b are constants. $(x + 2)$ is a factor of $f(x)$, and $f(1) = 18$.

(a) Find the values of a and b .

[4 marks]

Answer $a = \dots$, $b = \dots$

(b) Hence write $f(x)$ in the form $(x + 2)(px^2 + qx + r)$, where p , q and r are integers.

[2 marks]

Answer

- (c) Hence show that the equation $f(x) = 0$ has exactly one real root. [2 marks]

Total for D2: 8 marks

D3 The curve C has equation $y = x^3 + x^2 - 8x$. The line l has equation $y = 3x - 9$.

- (a) Show that the x -coordinates of the points where C meets l satisfy [1 mark]
 $x^3 + x^2 - 11x + 9 = 0$.

- (b) Use the factor theorem to show that $(x - 1)$ is a factor of [2 marks]
 $x^3 + x^2 - 11x + 9$.

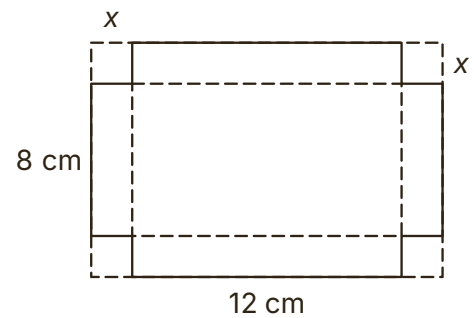
- (c) Hence find the x -coordinates of the other two points where C [4 marks]
meets l , giving your answers in the form $p \pm \sqrt{q}$.

Answer $x = \dots\dots$

Total for D3: 7 marks

- D4** An open box is made from a rectangular sheet of card 12 cm long and 8 cm wide. A square of side x cm is cut from each corner, and the sides are folded up along the dashed lines.

- (a) Show that the volume of the box, V cm³, is given by
 $V = 4x^3 - 40x^2 + 96x$. [2 marks]



Not drawn accurately

- (b) The volume of the box is 64 cm³. Show that
 $x^3 - 10x^2 + 24x - 16 = 0$. [1 mark]

- (c) Use the factor theorem to show that $(x - 2)$ is a factor of
 $x^3 - 10x^2 + 24x - 16$. Hence find all the solutions of
 $x^3 - 10x^2 + 24x - 16 = 0$, giving your answers in exact form. [4 marks]

Answer $x = \dots\dots$

- (d) Explain why only two of these solutions are possible sizes for the squares. [1 mark]

Total for D4: 8 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** $(x - 3)^2$ is a factor of $x^3 + px + q$, where p and q are constants. [7 marks]
Find the values of p and q , and find the third factor of $x^3 + px + q$.

Answer $p = \dots$, $q = \dots$;

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** $(x - 3)$ is a factor, so putting $x = 3$ gives $27 + 3p + q = 0$. That is one equation; you need a second.
- Hint 2** Divide $x^3 + 0x^2 + px + q$ by $(x - 3)$. The quotient is $x^2 + 3x + (9 + p)$, and because $(x - 3)$ is a factor TWICE, $(x - 3)$ is a factor of this quotient too.
- Hint 3** Put $x = 3$ in the quotient: $9 + 9 + 9 + p = 0$, so $p = -27$ and then $q = 54$. The quotient is $x^2 + 3x - 18 = (x - 3)(x + 6)$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use the factor theorem to show that a linear expression is a factor of a polynomial. ☐ ☐ ☐

I can find a factor by trying values, then factorise a cubic completely and sketch it. ☐ ☐ ☐

I can use given factors to find unknown constants, forming simultaneous equations if needed. ☐ ☐ ☐

I can use a factorised cubic to solve equations and decide how many real roots there are. ☐ ☐ ☐