The modulus function

112 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-the-modulus-function/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Solve the inequality $5 - 2x > 3x + 20$.

Answer .....

A2 Needed today [2 marks]

The line $y = 3x - 12$ meets the x-axis at A and the y-axis at B. Write down the coordinates of A and of B.

Answer .....

A3 Common slip [3 marks]

Given that $a = -5$ and $b = 3$, work out $|a + b|$, $|a| + |b|$ and $|a - b|$.

Answer .....

A4 Common slip [2 marks]

Find both values of x for which $|x - 3| = 5$.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

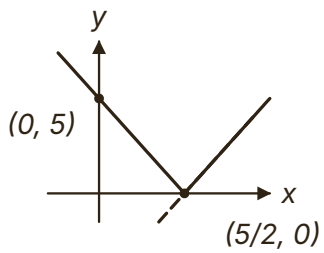
B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Sketch the graph of $y = |2x - 5|$. Give the coordinates of the points where the graph meets the axes.



Hint Sketch $y = 2x - 5$ first, then reflect the part below the x-axis in the x-axis.

B1 Sketch the graph of $y = |4 - 3x|$. Give the coordinates of the points where the graph meets the axes. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 2

Solve $|x - 4| = 2x + 1$.

Hint Solve $x - 4 = 2x + 1$ and $-(x - 4) = 2x + 1$, then put each answer back into the original equation.

B2 Solve $|2x + 3| = 3x - 1$. [5 marks]

Answer .....

EXAMPLE 3

Solve $|3x - 1| = |x + 5|$.

Hint Either the insides are equal or one is minus the other. Squaring both sides gives the same two answers.

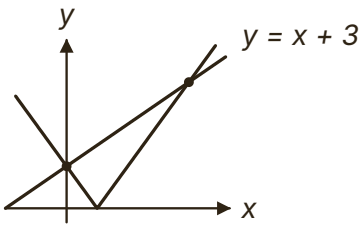
B3 Solve $|2x + 7| = |x - 4|$. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 4

Find the set of values of x for which $|2x - 3| < x + 3$.



Hint Find where the graphs of $y = |2x - 3|$ and $y = x + 3$ cross, then use a sketch to see where the V is below the line.

B4 Find the set of values of x for which $|x + 2| > 2x - 1$. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C1 Rosa solves $|x - 6| = 3x + 2$. She writes: "$x - 6 = 3x + 2$ gives $x = -4$, and $-(x - 6) = 3x + 2$ gives $x = 1$. So $x = -4$ or $x = 1$." Find her mistake and give the correct solution. [4 marks]

Answer .....

C2 Kai says that $|x + 3| = |x| + 3$ for every real number x. Show that Kai is wrong, and find the set of values of x for which his statement is true. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C3 Solve the inequality $|5 - 2x| \geq 9$. [4 marks]

Answer .....

C4 The graph of $y = |4x - a|$, where a is a positive constant, meets the y-axis at $(0, 12)$. (a) Find a and the coordinates of the vertex. (b) Solve $|4x - a| = x$. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C5 By squaring both sides, find the set of values of x for which $|x - 1| < |x + 5|$. [4 marks]

Answer .....

C6 Find the set of values of x for which $|2x + 1| \leq |x - 3|$. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D1 $f(x) = |2x - 8|$

(a) Sketch the graph of $y = f(x)$, giving the coordinates of any points where the graph meets the coordinate axes. [3 marks]

(b) Solve $f(x) = 12 - x$. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Hence find the set of values of x for which $f(x) > 12 - x$. [3 marks]

Answer .....

Total for D1: 9 marks

D2

> Sam solves $|3x + 1| < 2x + 6$. He writes: "$3x + 1 < 2x + 6$, so $x < 5$. Also $-(3x + 1) < 2x + 6$, so $-5x < 7$, giving $x < -\frac{7}{5}$. So the answer is $x < 5$."

Sam is wrong.

(a) Identify the mistake Sam makes. [2 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Find the correct set of values of x for which $|3x + 1| < 2x + 6$ [4 marks]

.

Answer .....

(c) Hence find how many integers satisfy $|3x + 1| < 2x + 6$. [2 marks]

Answer .....

Total for D2: 8 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D3 Solve the equation $|3x - 2| = |x| + 4$. [6 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D4 $g(x) = |x - 4|$. The line l has equation $y = kx + 2$, where k is a constant.

(a) Sketch the graph of $y = g(x)$, giving the coordinates of the points where it meets the axes. [2 marks]

(b) Find the value of k for which l passes through the vertex of the graph. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find the set of values of k for which the equation $|x - 4| = kx + 2$ has exactly two solutions. [4 marks]

Answer .....

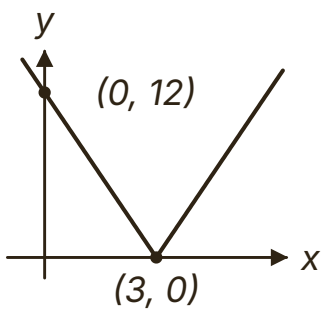
(d) State the number of solutions of $|x - 4| = kx + 2$ when $k = -2$. Give a reason. [2 marks]

Total for D4: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D5 The diagram shows part of the graph of $y = |ax + b|$, where a and b are constants and $a > 0$. The graph meets the y-axis at $(0, 12)$ and has its vertex at $(3, 0)$.

(a) Find the value of a and the value of b. [2 marks]



Answer .....

(b) Solve $|ax + b| = 2x + 3$. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find the set of values of the constant c for which $|ax + b| = 2x + c$ has two distinct solutions. [4 marks]

Answer .....

Total for D5: 9 marks

D6 $h(x) = |x + 2| + |x - 4|$

(a) Show that $h(x) = 6$ for $-2 \leq x \leq 4$. [2 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Write $h(x)$ without modulus signs for $x > 4$ and for $x < -2$. [2 marks]

Answer .....

(c) Solve $h(x) = 10$. [3 marks]

Answer .....

(d) Hence find the set of values of x for which $h(x) < 8$. [2 marks]

Answer .....

Total for D6: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The constant k satisfies $k \geq 0$. Find every value of k for which the equation $|2x - 3| = k|x + 1|$ has exactly one real solution. [9 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Because $k \geq 0$, $k|x+1| = |k(x+1)|$. So the equation means $2x - 3 = k(x+1)$ or $2x - 3 = -k(x+1)$.

Hint 2 The two cases give $x = \dfrac{3+k}{2-k}$ and $x = \dfrac{3-k}{2+k}$. When can there be only one solution?

Hint 3 Either the two answers are equal, or one of the cases has no solution at all. Look at $k = 0$ and at $k = 2$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can sketch the graph of y = |ax + b| and give its vertex and intercepts. ○ ○ ○
- I can solve an equation with a modulus and reject answers that do not work. ○ ○ ○
- I can solve an inequality with a modulus, using a sketch to choose the regions. ○ ○ ○
- I can explain why |a + b| is not always equal to |a| + |b|. ○ ○ ○

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.