The product and quotient rules

117 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-the-product-and-quotient-rules/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Differentiate $y = (3x - 2)^5$.

Answer .....

A2 Last lesson [2 marks]

Differentiate $y = e^{x^2+1}$.

Answer .....

A3 Needed today [3 marks]

Factorise fully $2x(x + 3)^4 + 4(x + 3)^3$.

Answer .....

A4 Common slip [2 marks]

Dev says the derivative of $x^2 \sin x$ is $2x \cos x$. Explain why he is wrong and give the correct derivative.

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Given $y = x^3 e^{2x}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and write it in factorised form.

Hint Let u = x cubed and v = e to the 2x. The derivative is u times the derivative of v, plus v times the derivative of u.

B1 Given $y = x^2 \ln(3x)$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and the gradient of the curve at the point where $x = \dfrac{1}{3}$. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 2

Given $y = (2x + 1)(x - 3)^4$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2(x - 3)^3(5x - 1)$.

Hint Use the product rule, with the chain rule for the bracket to the power 4. Then take out the common factor 2 times the bracket cubed.

B2 Given $y = x(2x - 1)^5$ [5 marks], show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (2x - 1)^4(12x - 1)$, and find the x-coordinates of the stationary points of the curve.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 3

Given $y = \dfrac{2x+1}{x-3}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

Hint With u on top and v underneath, the derivative is v times the derivative of u, minus u times the derivative of v, all over v squared. The order on top matters.

B3 [5 marks]

The curve C has equation $y = \dfrac{3x-2}{x^2+1}$. Find an equation of the tangent to C at the point where $x = 1$, in the form $y = mx + c$.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 4

Find the exact coordinates of the stationary point on the curve $y = \frac{e^{2x}}{x}$, $x \neq 0$.

Hint Use the quotient rule, take out the factor e to the 2x on top, then set the top equal to zero.

B4 Find the exact coordinates of the stationary point on the curve $y = \frac{\ln x}{x^2}$, $x > 0$. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 5

Find the gradient of the curve $y = x^2 \cos x$ at the point where $x = \pi$.

Hint Product rule, then put in x = pi. Remember sin pi is 0 and cos pi is minus 1.

B5 Find the x-coordinates of the stationary points on the curve $y = e^x \sin x$ for $0 \leq x \leq 2\pi$. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C1 [4 marks]

Ravi differentiates $y = \dfrac{x^2+1}{x-1}$ and writes $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(x^2+1)-2x(x-1)}{(x-1)^2}$. Explain his mistake, and find the x-coordinates of the stationary points of the curve.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C2 Find the exact coordinates of the stationary points on the curve $y = x^2 e^{-x}$ and decide the nature of each. [6 marks]

Answer .....

C3 Find the set of values of x for which $f(x) = \dfrac{x}{x^2 + 4}$ is increasing. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C4 Find the exact coordinates of the stationary point on the curve $y = \sqrt{x}\ln x$, $x > 0$. [5 marks]

Answer .....

C5 Given $y = (x-1)^2(x+2)^3$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (x-1)(x+2)^2(5x+1)$, and find the x-coordinates of the stationary points. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D1 The curve C has equation $y = (x^2 - 3)e^x$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (x^2 + 2x - 3)e^x$. [3 marks]

(b) Find the exact coordinates of the stationary points of C. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, and hence decide the nature of each stationary point. [4 marks]

Answer .....

Total for D1: 11 marks

D2

> Priya finds the stationary points of $y = \frac{x^2}{x-1}$. She differentiates the top and the bottom separately: "$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x}{1} = 2x$, so the only stationary point is $(0, 0)$."

Priya is wrong.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ correctly, in its simplest form. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Find the coordinates of both stationary points. [3 marks]

Answer .....

(c) Decide the nature of each stationary point. [3 marks]

Total for D2: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D3 The curve C has equation $y = \dfrac{2x - 1}{x^2 + 2}$. Find the exact coordinates of the points on C where the tangent is parallel to the x-axis. [6 marks]

Answer .....

D4 After a dose of a drug, the amount in a patient's blood, C mg per litre, is modelled by $C = 50te^{-0.2t}$, where t is the time in hours.

(a) Show that $\dfrac{\mathrm{d}C}{\mathrm{d}t} = 50e^{-0.2t}(1 - 0.2t)$. [3 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Find the greatest amount of the drug in the blood, and the time at which it happens. Give the amount to 3 significant figures. [3 marks]

Answer .....

(c) Find the rate of change of C when $t = 10$, to 3 significant figures. [2 marks]

Answer .....

(d) Explain what the sign of your answer to part (c) means in this context. [2 marks]

Total for D4: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D5 The curve C has equation $y = (x^2 + 1)\ln x$, $x > 0$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3 marks]

Answer .....

(b) Find an equation of the normal to C at the point where $x = 1$. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) The normal meets the y-axis at A. Find the area of the triangle formed by the normal and the two axes. [2 marks]

Answer .....

Total for D5: 9 marks

D6 The curve C has equation $y = e^{2x}\cos x$, $0 \leq x \leq \dfrac{\pi}{2}$.

(a) Show that at the stationary point $\tan x = 2$. [3 marks]

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Find the coordinates of the stationary point, to 3 significant figures. [4 marks]

Answer .....

Total for D6: 7 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [9 marks]

The curve C has equation $y = \dfrac{x^2 + k}{x - 1}$, $x \neq 1$, where k is a constant. Find the set of values of k for which C has two stationary points. When $k = 3$, find the stationary points and decide the nature of each.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Differentiate with the quotient rule. The bottom of your answer is never negative, so where can the gradient be zero?

Hint 2 The top is a quadratic in x. How many roots does it have, and when are there two? Could one of them be the excluded value x = 1?

Hint 3 For $k = 3$ the top is $(x - 3)(x + 1)$. Look at its sign just below and just above each root.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can differentiate a product with the product rule.	○ ○ ○
I can differentiate a quotient with the quotient rule, with the terms on top in the right order.	○ ○ ○
I can combine these with the chain rule and factorise the answer.	○ ○ ○
I can use the derivative to find tangents, normals and stationary points.	○ ○ ○

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.