# The trapezium rule

113 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-the-trapezium-rule/

Date

## A Do Now

Quick questions to start.

**A1** Needed today [2 marks]

$f(x) = \ln x$, $x > 0$. Find $f''(x)$ and say whether the curve $y = \ln x$ is concave or convex.

Answer .....

**A2** Common slip [2 marks]

Work out $0.6 \sin 0.6$, correct to $4$ decimal places.

Answer .....

**A3** Common slip [2 marks]

The interval $1 \leq x \leq 3$ is split into $4$ strips of equal width. Write down the width of each strip and the $x$-values at the edges of the strips.

Answer .....

**A4** Needed today [2 marks]

A trapezium has parallel sides of length $2.4$ and $3.6$, and the distance between them is $0.5$. Find its area.

Answer .....



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Your teacher works through each example with you. Then try the one beside it.

**EXAMPLE 1**

Use the trapezium rule with $4$ strips to estimate $\int_0^2 \sqrt{1+x^2}\,dx$, giving your answer to $3$ decimal places.

The table shows values of $y = \sqrt{1+x^2}$.

| x | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| y | 1 | 1.118 | 1.4142 | 1.8028 | 2.2361 |

**Hint** $h = \frac{2-0}{4} = 0.5$. Work out the $5$ ordinates, then use $\frac{h}{2}\left[y_0 + y_4 + 2(y_1 + y_2 + y_3)\right]$.

**B1** Use the trapezium rule with $4$ strips to estimate $\int_0^1 e^{x^2}\,dx$, giving your answer to $3$ decimal places. [4 marks]

Answer .....



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**EXAMPLE 2**

Use the trapezium rule with $4$ strips to estimate $\displaystyle\int_1^2 \frac{1}{x}\,dx$. Say whether your estimate is too big or too small, and find the percentage error.

**Hint** $f''(x) = \dfrac{2}{x^3} > 0$, so the curve is convex. Each chord lies above the curve. The exact value is $\ln 2$.

**B2** [6 marks]

Use the trapezium rule with $3$ strips to estimate $\displaystyle\int_1^4 \sqrt{x}\,dx$. Explain whether your estimate is too big or too small. Find the exact value and the percentage error.

Answer .....


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**EXAMPLE 3**

Use the trapezium rule with $4$ strips to estimate $I = \int_0^4 e^{\sqrt{x}}\,dx$. Hence estimate $\int_0^4 \left(e^{\sqrt{x}} + 3x\right) dx$ and $\int_0^4 e^{\sqrt{x}+1}\,dx$.

**Hint** The extra $3x$ can be integrated exactly. And $e^{\sqrt{x}+1} = e \times e^{\sqrt{x}}$.

**B3** [5 marks]

Use the trapezium rule with $4$ strips to estimate $I = \int_0^2 \ln(1+x^2)\,dx$.

Hence estimate $\int_0^2 \ln\left((1+x^2)^3\right) dx$ and $\int_0^2 \ln\left(e(1+x^2)\right) dx$.

Answer .....



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**EXAMPLE 4**

Use the trapezium rule with $4$ strips to estimate $\displaystyle\int_0^{\pi/2} x \sin x \, dx$. Find the exact value by integration by parts, and say how the estimate could be made more accurate.

**Hint** $h = \dfrac{\pi}{8}$. Keep the calculator in radians.

**B4** [5 marks]

Use the trapezium rule with $4$ strips to estimate $\displaystyle\int_0^{\pi/2} x \cos x \, dx$. Find the exact value and the percentage error.

Answer .....



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**C1** [5 marks]

Mia uses the trapezium rule with $4$ strips to estimate $\int_0^2 2^x \, dx$. She works out the ordinates at $x = 0, 0.5, 1, 1.5, 2$ correctly. Then she says: "There are $5$ ordinates, so $h = \frac{2}{5} = 0.4$," and gets $3.497$.

Explain her mistake. Find the correct estimate and say whether it is too big or too small.

Answer .....



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**C2** Josh uses the trapezium rule with $2$ strips to estimate $\int_0^2 e^x \, dx$. He says: "The curve is increasing, so the estimate must be too small." Find his estimate and the exact value, and explain why he is wrong. [5 marks]

Answer .....


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**C3** Ella uses the trapezium rule with $3$ strips to estimate $\int_0^{1.2} x \sin x \, dx$. Her table of values of $y = x \sin x$ is shown. She gets $0.0106$. Explain what has gone wrong and find the correct estimate. [5 marks]

| x | 0 | 0.4 | 0.8 | 1.2 |
|---|---|---|---|---|
| y | 0 | 0.0028 | 0.0112 | 0.0251 |

Answer .....


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**C4** $\int_0^1 \frac{4}{1+x^2}\,dx = \pi$. Use the trapezium rule with $2$ strips and then with $4$ strips to estimate the integral. Find the percentage error in each, to $2$ decimal places, and comment on what happens when the number of strips is doubled. [6 marks]

Answer .....

## D Exam-style questions

Edexcel exam style

**D1** The table shows values of $y = x \ln x$ for $1 \leq x \leq 3$, to $4$ decimal places.

**(a)** Complete the table. [2 marks]

| x | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|
| y | 0 | ? | 1.3863 | ? | 3.2958 |

Answer .....


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**(b)** Use the trapezium rule with all the values in the table to estimate $\int_1^3 x \ln x \, dx$, giving your answer to $3$ decimal places. [3 marks]

Answer .....

**(c)** Use integration by parts to find the exact value of $\int_1^3 x \ln x \, dx$, in the form $a \ln 3 + b$. [4 marks]

Answer .....


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**(d)** Find the percentage error in your answer to part (b), and explain why the estimate is too big. [2 marks]

Answer .....

**Total for D1: 11 marks**

**D2**

Amir uses the trapezium rule with $4$ strips to estimate $\int_0^2 \sqrt{4+x^2}\,dx$.

He writes: "$0.25\,[2(2)+2(2.8284)+2.0616+2.2361+2.5]=4.114$. The curve bends upwards, so this is an underestimate."

Amir is wrong twice.

**(a)** Explain the mistake in his use of the trapezium rule. [2 marks]


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**(b)** Find the correct estimate, to $3$ decimal places. [3 marks]

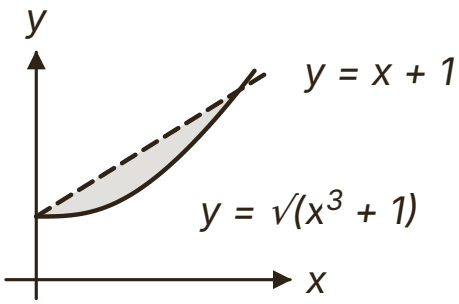
Answer .....

**(c)** Given that $\frac{d^2}{dx^2}\sqrt{4+x^2} = \frac{4}{(4+x^2)^{3/2}}$, explain whether the correct estimate is too big or too small. [4 marks]

**Total for D2: 9 marks**


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**D3** The diagram shows the line $y = x + 1$ and the curve $y = \sqrt{x^3 + 1}$. [7 marks]
They meet at $x = 0$ and at $x = 2$. Use the trapezium rule with $4$ strips to estimate the area of the shaded region between them, giving your answer to $2$ decimal places.




Answer .....


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**D4** A cyclist starts from rest. Her speed $v$ m s$^{-1}$ is recorded every $2$ seconds for $10$ seconds.

| t (s) | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| v (m s$^{-1}$) | 0.00 | 6.59 | 11.01 | 13.98 | 15.96 | 17.29 |

**(a)** Use the trapezium rule with all the values in the table to estimate the distance she travels in the $10$ seconds. [3 marks]

Answer .....

**(b)** The speeds were taken from the model $v = 20\left(1 - e^{-0.2t}\right)$. Show that the model gives a distance of $100 + 100e^{-2}$ m in the $10$ seconds. [4 marks]


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**(c)** Explain why your answer to part (a) is smaller than the model's distance. [2 marks]

**Total for D4: 9 marks**

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**D5** The curve $y = \sqrt{1 - x^2}$ for $0 \leq x \leq 1$ is a quarter of a circle of radius $1$.

**(a)** Complete the table, to $4$ decimal places. [2 marks]

| x | 0 | 0.25 | 0.5 | 0.75 | 1 |
|---|---|---|---|---|---|
| y | 1 | ? | 0.866 | ? | 0 |

Answer .....


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**(b)** Use the trapezium rule with all the values in the table to estimate $\int_0^1 \sqrt{1-x^2}\,dx$. [3 marks]

Answer .....

**(c)** Explain why the exact value of the integral is $\frac{\pi}{4}$, and use your answer to part (b) to estimate $\pi$. [2 marks]

Answer .....


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**(d)** State whether this estimate of $\pi$ is too big or too small, giving a reason, and say how to make it more accurate. [2 marks]

**Total for D5: 9 marks**

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**D6** $f(x) = \dfrac{3}{(x+1)(x+4)}, x \geq 0.$

**(a)** Express $f(x)$ in partial fractions. [3 marks]

Answer .....


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**(b)** Hence show that $\int_0^2 f(x)\,dx = \ln 2$. [3 marks]

**(c)** Use the trapezium rule with $2$ strips to estimate the integral, and find the percentage error, to $2$ significant figures. [3 marks]

Answer .....

**Total for D6: 9 marks**


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## E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

**E1** [10 marks]

Let $T_n$ be the trapezium rule estimate of $\displaystyle\int_0^1 e^x\,dx$ using $n$ strips. Show that $T_n = \dfrac{e-1}{2n} \times \dfrac{e^{1/n}+1}{e^{1/n}-1}$. Hence find the smallest $n$ for which $T_n$ is more than the exact value by less than $0.002$.



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## Extension hints

Cover these with paper. Uncover one at a time.

**E1**

**Hint 1** The strips have width $h = \dfrac{1}{n}$, and the ordinates are $e^0, e^{1/n}, e^{2/n}, \ldots, e^1$.

**Hint 2** Write $T_n = \dfrac{h}{2}(S + S')$, where $S = \displaystyle\sum_{k=0}^{n-1} e^{k/n}$ and $S' = \displaystyle\sum_{k=1}^{n} e^{k/n} = e^{1/n}S$. $S$ is a geometric series.

**Hint 3** The exact value is $e - 1$. Work out $T_n - (e - 1)$ for a few values of $n$ near where it drops below $0.002$.

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

| | |
|---|---|
| I can use the trapezium rule with n strips, taking h as the width of one strip. | ○ ○ ○ |
| I can say whether a trapezium rule estimate is too big or too small from whether the curve is convex or concave. | ○ ○ ○ |
| I can find the percentage error against an exact answer and say how to make an estimate better. | ○ ○ ○ |
| I can use one trapezium rule estimate to estimate a related integral. | ○ ○ ○ |


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