



Transforming graphs

70 marks

Date _____

A Do Now

Quick questions to start.

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|--|--|
| <p>A1 Needed today [2 marks]</p> <p>Factorise fully
$x^3 - 7x^2 + 12x$.</p> | <p>A2 Last lesson [2 marks]</p> <p>Find the coordinates of
the points where $y = \frac{3}{x}$
meets $y = x + 2$.</p> |
|--|--|

Answer

Answer

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|---|---|
| <p>A3 From chapter 2 [2 marks]</p> <p>Write $x^2 + 14x + 43$ in the
form $(x + p)^2 + q$.</p> | <p>A4 Needed today [2 marks]</p> <p>$f(x) = 7x - x^2$. Write
$f(x + 1)$ in the form
$ax^2 + bx + c$.</p> |
|---|---|

Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$f(x) = (x + 1)(x - 3)^2$. On the same axes sketch $y = f(x)$ and $y = f(x + 2)$, showing where each curve meets the axes.

Hint $f(x + 2)$ moves the curve 2 to the left. Write it as $(x + 3)(x - 1)^2$ to find the new y -intercept.

B1 $g(x) = (2 - x)(x + 3)^2$. [4 marks]
Sketch $y = g(x - 1)$, showing where the curve meets the axes.

EXAMPLE 2

On separate axes sketch $y = \frac{4}{x} + 2$ and $y = \frac{4}{x + 2}$. Give the equations of the asymptotes and the coordinates of any point where each curve meets an axis.

Hint $+2$ outside moves $y = \frac{4}{x}$ up 2; $x + 2$ inside moves it 2 left. Each asymptote moves with the curve.

B2 On separate axes [5 marks]
sketch $y = \frac{9}{x^2} - 1$ and $y = \frac{9}{(x - 1)^2}$. Give the equations of the asymptotes and the coordinates of any point where each curve meets an axis.

EXAMPLE 3

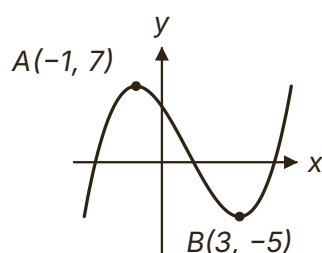
$f(x) = (x + 4)(x - 2)$, whose minimum point is $(-1, -9)$. On separate axes sketch $y = f(2x)$ and $y = -f(x)$, showing where each meets the axes and its turning point.

Hint $f(2x)$ halves every x -coordinate and keeps every y . $-f(x)$ keeps every x and changes the sign of every y .

B3 $g(x) = (x - 1)(x + 7)$, [4 marks]
whose minimum point is $(-3, -16)$. On separate axes sketch $y = 3g(x)$ and $y = g(-x)$, showing where each meets the axes and its turning point.

EXAMPLE 4

The diagram shows $y = f(x)$, which has a maximum at $A(-1, 7)$ and a minimum at $B(3, -5)$. Find where A and B go on (a) $y = f(\frac{1}{2}x)$ (b) $y = 3 - f(x)$, saying which is now the maximum.



Hint Write $3 - f(x)$ as $-f(x) + 3$: reflect in the x -axis first, then move up 3.

B4 $P(-3, 4)$ and $Q(1, -2)$ [4 marks]
lie on $y = f(x)$. Find where P and Q go on (a) $y = f(3x)$ (b) $2y = f(x) - 6$.

Answer

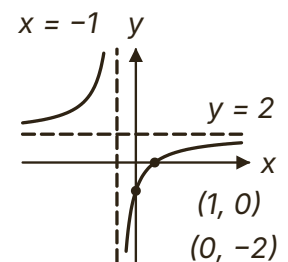
- C1** The point $(6, -12)$ on $y = f(x)$ goes to $(2, -4)$ on $y = pf(qx)$ and to $(9, -7)$ on $y = f(x - r) + s$. Find p, q, r and s . [4 marks]

Answer

- C2** The curve $y = \frac{a}{x+b} + c$ has asymptotes $x = -3$ and $y = 2$, and passes through the point $(-1, 6)$. Find the values of a, b and c . [3 marks]

Answer

- C3** The diagram shows the curve $y = f(x)$. It has asymptotes $x = -1$ and $y = 2$, and it passes through $(1, 0)$ and $(0, -2)$. On separate axes sketch $y = f(2x)$ and $y = -f(x)$. Give the equations of the asymptotes and the coordinates of the points where each curve meets the axes. [5 marks]



D1 $f(x) = x^2 - 16x + 48.$

- (a) Write $f(x)$ in the form $(x - p)^2 - q$. [2 marks]

Answer

- (b) Hence describe fully the single transformation that maps the curve $y = x^2$ onto the curve $y = f(x)$. [2 marks]

Answer

- (c) Hence sketch $y = f(2x)$, showing the coordinates of its minimum point and of the points where it meets the axes. [3 marks]

- (d) The curve $y = f(x) + k$ meets the x -axis at exactly one point. [2 marks]
Find the value of k .

Answer

Total for D1: 9 marks

D2

The curve $y = f(x)$ has one turning point, a minimum at $(4, -3)$. Josh says: " $y = 2f(x) + 5$ is $y = f(x)$ moved up 5 and then stretched by scale factor 2 parallel to the y -axis, so its minimum point is $(4, 4)$."

Josh is wrong.

- (a) Write down the equation of the curve that Josh's description gives, and find its minimum point. [2 marks]

Answer

- (b) Hence explain Josh's mistake, and find the minimum point of $y = 2f(x) + 5$. [3 marks]

Answer

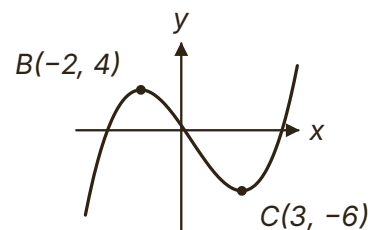
Total for D2: 5 marks

- D3** $f(x) = (x - 4)(x + 2)^2$, and $y = f(x)$ has a minimum point at $(2, -32)$. The curve $y = f(x + p) + q$ touches the x -axis at the origin. Find the two possible pairs of values of p and q . [5 marks]

Answer

D4 The diagram shows the curve $y = f(x)$. It has exactly two turning points: a maximum at $B(-2, 4)$ and a minimum at $C(3, -6)$. As $x \rightarrow \infty, y \rightarrow \infty$, and as $x \rightarrow -\infty, y \rightarrow -\infty$.

- (a) Find where B and C go on $y = -2f(x)$, saying which is now the maximum. [2 marks]



Answer

- (b) Find where B and C go on $y = f(3x) + 2$. [2 marks]

Answer

- (c) Find the set of values of k for which $f(x) = k$ has exactly three real solutions. [2 marks]

Answer

- (d) Hence find the set of values of c for which $2f(x) + c = 0$ has exactly three real solutions. [2 marks]

Answer

Total for D4: 8 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Describe a sequence of transformations that maps the curve $y = \frac{1}{x}$ onto the curve $y = \frac{3x - 1}{x + 2}$. Hence sketch $y = \frac{3x - 1}{x + 2}$, giving its asymptotes and the points where it meets the axes. [6 marks]

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write the top as $3(x + 2) - 7$.
Hint 2 So $y = 3 - \frac{7}{x + 2}$. Build it from $\frac{1}{x}$ one change at a time: $\frac{1}{x + 2}$, then $\frac{7}{x + 2}$, then $-\frac{7}{x + 2}$, then add 3.
Hint 3 $3x - 1 = 3(x + 2) - 7$, so $y = 3 - \frac{7}{x + 2}$. A translation by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ gives $\frac{1}{x + 2}$; a stretch parallel to the y -axis, scale factor 7, gives $\frac{7}{x + 2}$; a reflection in the x -axis gives $-\frac{7}{x + 2}$; a translation by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ gives the curve. The asymptotes move with it, to $x = -2$ and $y = 3$. Put $y = 0$: $x = \frac{1}{3}$. Put $x = 0$: $y = -\frac{1}{2}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can sketch $y = f(x + a)$, $y = f(x) + a$, $y = af(x)$ and $y = f(ax)$. ☐ ☐ ☐

I can find where a point goes when a graph is transformed. ☐ ☐ ☐

I can find the new asymptotes of a transformed graph. ☐ ☐ ☐

I can find the constants of a transformation. ☐ ☐ ☐