



Equations and identities

76 marks

Date _____

A Do Now

Quick questions to start.

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| <p>A1 Needed today [2 marks]</p> <p>Solve $3x^2 + 7x - 6 = 0$.</p> <p>Answer</p> | <p>A2 Last lesson [2 marks]</p> <p>Solve $\tan x = -2.5$ for $0^\circ \leq x < 360^\circ$. Give your answers to 1 decimal place.</p> <p>Answer</p> |
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| <p>A3 Needed today [2 marks]</p> <p>Write $4 \cos^2 x + 3 \sin x - 2$ in terms of $\sin x$ only.</p> <p>Answer</p> | <p>A4 From chapter 2 [2 marks]</p> <p>Use the discriminant to show that $x^2 - 3x + 5 = 0$ has no real roots.</p> <p>Answer</p> |
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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it. Give angles to 1 decimal place where they are not exact.

EXAMPLE 1

Solve $3 \cos^2 \theta + 5 \cos \theta - 2 = 0$ for $0^\circ \leq \theta < 360^\circ$.

Hint This is $3c^2 + 5c - 2 = 0$ with $c = \cos \theta$. Factorise, then solve each factor.

B1 Solve [3 marks]

$4 \sin^2 x + 11 \sin x - 3 = 0$
for $-180^\circ \leq x \leq 180^\circ$.

Answer $x = \dots\dots$

EXAMPLE 2

Solve $\tan^2(\theta + 20^\circ) = 3$ for $0^\circ \leq \theta < 360^\circ$.

Hint Square root both sides: there are two equations, $\tan(\theta + 20^\circ) = \sqrt{3}$ and $\tan(\theta + 20^\circ) = -\sqrt{3}$. The interval for $\theta + 20^\circ$ is 20° to 380° .

B2 Solve [4 marks]

$4 \cos^2(x - 15^\circ) = 3$ for $0^\circ \leq x < 360^\circ$. Give your answers exactly.

Answer $x = \dots\dots$

EXAMPLE 3

Solve $2 \cos^2 \theta + 3 \sin \theta = 3$ for
 $0^\circ \leq \theta < 360^\circ$.

Hint Replace $\cos^2 \theta$ with $1 - \sin^2 \theta$ to get a quadratic in $\sin \theta$.

B3 Solve [5 marks]

$6 \sin^2 x + \cos x - 4 = 0$
 for $0^\circ \leq x < 360^\circ$.

Answer $x = \dots\dots$

EXAMPLE 4

Solve $5 \sin \theta \cos \theta = 2 \cos \theta$ for
 $0^\circ \leq \theta < 360^\circ$.

Hint Do not divide by $\cos \theta$: it can be 0.
 Move everything to one side and take out the common factor $\cos \theta$.

B4 Solve [4 marks]

$7 \sin x \tan x = 3 \tan x$
 for $0^\circ \leq x < 360^\circ$.

Answer $x = \dots\dots$

c Practice

A-Level

C1 Solve $\sin^2 \theta + 3 \sin \theta - 1 = 0$ for $0^\circ \leq \theta < 360^\circ$. [4 marks]

Answer $\theta = \dots\dots$

C2 Find the exact solutions of $3 \tan \theta = 2 \cos \theta$ for $-180^\circ \leq \theta \leq 180^\circ$. [4 marks]

Answer $\theta = \dots\dots$

C3 Find the exact solutions of $2 \cos^2 2x = \cos 2x$ for $0^\circ \leq x < 180^\circ$. [4 marks]

Answer $x = \dots\dots$

C4 Solve $\frac{3 \sin^2 \theta}{1 - \cos \theta} = 2$ for $0^\circ < \theta < 360^\circ$. [4 marks]

Answer $\theta = \dots\dots$

D1 Ravi is asked to solve $5 \sin^2(x + 25^\circ) = 2$ for $0^\circ \leq x < 360^\circ$. Ravi writes: " $\sin(x + 25^\circ) = \sqrt{0.4} = 0.6325$, so $x + 25^\circ = 39.2^\circ$ or 140.8° . So $x = 14.2^\circ$ or $x = 115.8^\circ$."

Ravi has not found every solution.

(a) Explain the mistake Ravi has made. [2 marks]

(b) Hence, or otherwise, find all the solutions, giving your answers to 1 decimal place. [4 marks]

Answer $x = \dots\dots$

Total for D1: 6 marks

D2 The angle θ is in the interval $0^\circ \leq \theta < 360^\circ$.

(a) Show that the equation $5 \cos^2 \theta + 3 \sin \theta = 4$ can be written as $5 \sin^2 \theta - 3 \sin \theta - 1 = 0$. [2 marks]

(b) Hence show that $\sin \theta = \frac{3 \pm \sqrt{29}}{10}$. [2 marks]

(c) Hence solve $5 \cos^2 \theta + 3 \sin \theta = 4$ for $0^\circ \leq \theta < 360^\circ$, giving your answers to 1 decimal place. [4 marks]

Answer $\theta = \dots\dots$

Total for D2: 8 marks

D3 The angle x is in the interval $0^\circ \leq x \leq 180^\circ$.

(a) Show that the equation $4 \sin 2x \tan 2x = 15$ can be written as $4 \cos^2 2x + 15 \cos 2x - 4 = 0$. [3 marks]

- (b) Hence solve $4 \sin 2x \tan 2x = 15$ for $0^\circ \leq x \leq 180^\circ$, giving your answers to 1 decimal place. [5 marks]

Answer $x = \dots\dots$

Total for D3: 8 marks

D4 The angle θ is in the interval $0^\circ \leq \theta < 360^\circ$.

- (a) Show that the equation $3 \cos^2 \theta - 2 \sin \theta = 5$ has no solutions. [3 marks]

- (b) Solve $3 \cos^2 \theta - 2 \sin \theta = 2$ for $0^\circ \leq \theta < 360^\circ$, giving your answers to 1 decimal place where they are not exact. [5 marks]

Answer $\theta = \dots\dots$

Total for D4: 8 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Find every solution of $\sin^4 \theta - \cos^4 \theta = \frac{1}{2}$ in the interval $0^\circ \leq \theta < 360^\circ$. Give your answers exactly. [6 marks]

Answer $\theta = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** $\sin^4 \theta - \cos^4 \theta$ is a difference of two squares.
Hint 2 It is $(\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta)$, and the first bracket is 1.
Hint 3 Replace $\cos^2 \theta$ with $1 - \sin^2 \theta$:
 $2 \sin^2 \theta - 1 = \frac{1}{2}$, so $\sin \theta = \pm \frac{\sqrt{3}}{2}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can solve a quadratic in $\sin$, $\cos$ or $\tan$, and reject a value that is impossible. ☐ ☐ ☐

I can use $\sin^2 \theta + \cos^2 \theta = 1$ or $\tan \theta = \sin \theta / \cos \theta$ to get an equation in one ratio. ☐ ☐ ☐

I can take out a common factor instead of dividing, so no solutions are lost. ☐ ☐ ☐

I can show that an equation has no solutions. ☐ ☐ ☐