



Trigonometric identities

70 marks

Date _____

A Do Now

Quick questions to start.

- A1** [Last lesson] [2 marks] Find the exact value of $\cos 150^\circ$.
- A2** [Needed today] [2 marks] Factorise fully $16x^4 - 81$.

Answer

Answer

- A3** [From chapter 9] [2 marks] A triangle has sides 5 cm, 7 cm and 8 cm. Find the angle opposite the 7 cm side.
- A4** [From chapter 7] [2 marks] Write $\frac{1}{x} - x$ as a single fraction.

Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Given that $\cos \theta = -\frac{8}{17}$ and that θ is reflex, find the exact values of $\sin \theta$ and $\tan \theta$.

Hint Find $\sin^2 \theta$ from $\sin^2 \theta + \cos^2 \theta = 1$. Then decide the sign: a reflex angle with a negative cosine is in the third quadrant.

B1 Given that $\sin \alpha = \frac{3}{7}$ [4 marks]
and that α is obtuse,
find the exact values of
 $\cos \alpha$ and $\tan \alpha$. Give
each answer with a
rational denominator.

Answer $\cos \alpha = \dots\dots$ $\tan \alpha = \dots\dots$

EXAMPLE 2

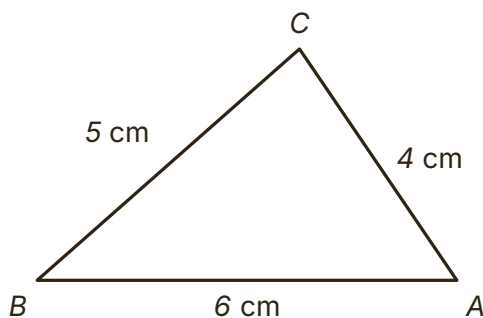
Prove that $\frac{\cos \theta}{1 + \sin \theta} + \tan \theta \equiv \frac{1}{\cos \theta}$.

Hint Start from the left-hand side. Write $\tan \theta$ as $\frac{\sin \theta}{\cos \theta}$, then put the two terms over the common denominator $\cos \theta(1 + \sin \theta)$.

B2 Prove that [4 marks]
 $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} \equiv \frac{2}{\sin \theta}$.

EXAMPLE 3

In triangle ABC , $AB = 6$ cm, $BC = 5$ cm and $AC = 4$ cm. Find the exact values of $\cos B$ and $\sin B$.

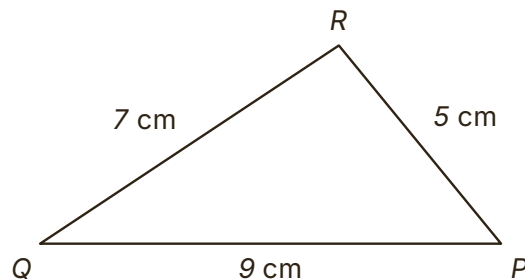


Not drawn accurately

Hint The cosine rule gives $\cos B$ exactly.
Angle B is in a triangle, so $\sin B$ is positive.

B3 [6 marks]

Triangle PQR has $PQ = 9$ cm, $QR = 7$ cm, $PR = 5$ cm. Find $\cos Q$ and $\sin Q$ exactly, then the exact area.



Not drawn accurately

Answer $\sin Q = \dots$ area = $\dots$

c Practice

A-Level

C1 Simplify (a) $8 - 8 \cos^2 4\theta$ (b) $\frac{\sin^2 \theta}{\cos \theta - \cos^3 \theta}$

[3 marks]

Answer (a) (b)

C2 $21 \tan \theta + 20 = 0$ and $90^\circ < \theta < 180^\circ$. Find $\sin \theta$ and $\cos \theta$ exactly. [3 marks]

Answer $\sin \theta = \dots\dots$ $\cos \theta = \dots\dots$

C3 $2 \sin x \cos y - \cos x \sin y = 4 \cos x \cos y$. Find $\tan y$ in terms of $\tan x$. [3 marks]

Answer $\tan y = \dots\dots$

C4 Prove that $\tan^2 \theta - \sin^2 \theta \equiv \tan^2 \theta \sin^2 \theta$. [3 marks]

D Exam-style questions

Edexcel exam style

D1

$$\cos \theta = -\frac{2}{3}, 180^\circ < \theta < 270^\circ. \text{ Mia: "sin } \theta = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}, \text{ so}$$
$$\tan \theta = -\frac{\sqrt{5}}{2}."$$

Mia's value of $\tan \theta$ is wrong.

(a) Explain the mistake Mia has made. [1 mark]

(b) Find the correct exact value of $\tan \theta$.

[2 marks]

Answer $\tan \theta = \dots\dots$

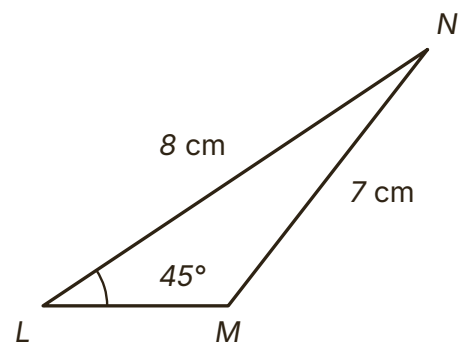
(c) Find the exact value of $\frac{1 - \sin \theta}{1 + \sin \theta}$, giving your answer in the form $\frac{a + b\sqrt{5}}{2}$, where a and b are integers. [3 marks]

Answer

Total for D1: 6 marks

D2 The diagram shows triangle LMN with $LN = 8$ cm, $MN = 7$ cm and angle $MLN = 45^\circ$. Angle LMN is obtuse.

(a) Show that $\sin M = \frac{4\sqrt{2}}{7}$. [3 marks]



Not drawn accurately

(b) Find the exact value of $\cos M$.

[2 marks]

Answer $\cos M = \dots\dots$

- (c) Hence, using the cosine rule in angle M , find the exact length of LM . Give your answer in the form $a\sqrt{2} - \sqrt{b}$, where a and b are integers. [4 marks]

Answer $LM = \dots\dots$ cm

Total for D2: 9 marks

D3 $f(\theta) = \frac{\sin^2 \theta}{1 - \cos \theta}$, where $\cos \theta \neq 1$.

- (a) Prove that $f(\theta) \equiv 1 + \cos \theta$, where $\cos \theta \neq 1$. [3 marks]

- (b) Hence find the exact value of $f(150^\circ)$. [2 marks]

Answer

- (c) Hence find the set of values that $f(\theta)$ can take. Explain why $f(\theta)$ can never equal 2. [2 marks]

Answer $\leq f(\theta) <$

Total for D3: 7 marks

D4 Given that $p = 3 \sin \theta + \cos \theta$ and $q = \sin \theta - 3 \cos \theta$,

- (a) show that $p^2 + q^2 = 10$. [3 marks]

- (b) Given also that $q = -1$ and $p > 0$, find the value of p . [2 marks]

Answer $p =$

- (c) Hence find the exact values of $\sin \theta$ and $\cos \theta$. [3 marks]

Answer $\sin \theta =$ $\cos \theta =$

Total for D4: 8 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The angle θ is acute and $\tan \theta + \frac{1}{\tan \theta} = 4$. Find the exact value of $\sin \theta + \cos \theta$. Given also that $\theta > 45^\circ$, find the exact value of $\sin \theta$. [6 marks]

Answer $\sin \theta + \cos \theta = \dots\dots$ $\sin \theta = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1 Hint 1** Write $\tan \theta$ as $\frac{\sin \theta}{\cos \theta}$ and add the two fractions. Then think about $(\sin \theta + \cos \theta)^2$ and $(\sin \theta - \cos \theta)^2$.
- Hint 2** $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta}$, so $\sin \theta \cos \theta = \frac{1}{4}$.
- Hint 3** $(\sin \theta + \cos \theta)^2 = 1 + 2 \times \frac{1}{4} = \frac{3}{2}$, and the sum is positive, so it is $\frac{\sqrt{6}}{2}$.
 $(\sin \theta - \cos \theta)^2 = 1 - \frac{1}{2} = \frac{1}{2}$, and $\sin \theta > \cos \theta$ when $\theta > 45^\circ$, so $\sin \theta - \cos \theta = \frac{\sqrt{2}}{2}$. Adding and halving:
 $\sin \theta = \frac{\sqrt{6} + \sqrt{2}}{4}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use sin squared plus cos squared equals 1 and tan equals sin over cos to simplify.



I can prove a trigonometric identity, starting from one side.



I can find an exact ratio from another one and the quadrant.



I can eliminate an angle from two equations.

