

Vectors and representing vectors

70 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-vectors-and-representing-vectors/

Date _____

A Do Now

Quick questions to start.

- | | |
|--|--|
| <p>A1 Needed today [2 marks]</p> <p>Simplify
$3(2x - y) - 2(x - 4y)$.</p> | <p>A2 From chapter 3 [2 marks]</p> <p>Solve the simultaneous equations $2p + q = 7$ and $p - 3q = 14$.</p> |
|--|--|

Answer

Answer $p = \dots\dots$ $q = \dots\dots$

- | | |
|--|---|
| <p>A3 From chapter 5 [2 marks]</p> <p>The line through $(1, 2)$ and $(5, k)$ is parallel to the line $y = 3x + 1$. Find k.</p> | <p>A4 Needed today [2 marks]</p> <p>The point P divides the line AB in the ratio $AP : PB = 3 : 2$.
$AB = 15$ cm. Find AP.</p> |
|--|---|

Answer $k = \dots\dots$

Answer $AP = \dots\dots$ cm

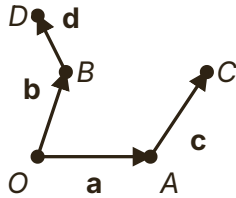
B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

In the diagram, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{AC} = \mathbf{c}$ and $\overrightarrow{BD} = \mathbf{d}$. Find, in terms of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$, (a) $\overrightarrow{AB}$ (b) $\overrightarrow{BC}$ (c) $\overrightarrow{CD}$.

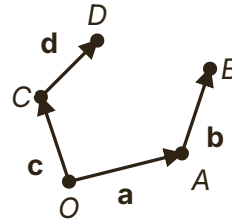


Not drawn accurately

Hint Go along vectors you know. Going against an arrow gives a minus sign:

$$\overrightarrow{AO} = -\mathbf{a}.$$

B1 Using the vectors marked in the diagram, find (a) $\overrightarrow{CA}$ (b) $\overrightarrow{BC}$ (c) $\overrightarrow{DB}$. [3 marks]



Not drawn accurately

Answer (a) (b) (c)

EXAMPLE 2

$\mathbf{a} = 4\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = -3\mathbf{i} + 6\mathbf{j}$. (a) Find $3\mathbf{a} + 2\mathbf{b}$ in $\mathbf{i}$, $\mathbf{j}$ form. (b) Find $\mathbf{a} - 2\mathbf{b}$ as a column vector. (c) Show that $3\mathbf{a} + 2\mathbf{b}$ is parallel to $\mathbf{i} + 3\mathbf{j}$. (d) Find λ if $\mathbf{a} + \lambda\mathbf{b}$ is parallel to $\mathbf{j}$.

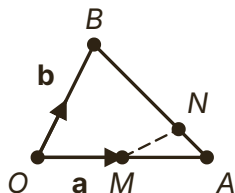
Hint Work on the $\mathbf{i}$ parts and the $\mathbf{j}$ parts separately. Two vectors are parallel when one is a number times the other. A vector parallel to $\mathbf{j}$ has no $\mathbf{i}$ part.

B2 $\mathbf{c} = 5\mathbf{i} - 3\mathbf{j}$ and $\mathbf{d} = -3\mathbf{i} + \mathbf{j}$. (a) Find $2\mathbf{c} + 3\mathbf{d}$ in $\mathbf{i}$, $\mathbf{j}$ form. (b) Find $\mathbf{c} - 4\mathbf{d}$ as a column vector. (c) Show that $2\mathbf{c} + 3\mathbf{d}$ is parallel to $-2\mathbf{i} + 6\mathbf{j}$. [4 marks]

Answer (a) (b)

EXAMPLE 3

OAB is a triangle with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of OA and N is the point on AB with $AN : NB = 1 : 3$. Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, (a) $\overrightarrow{AB}$ (b) $\overrightarrow{AN}$ (c) $\overrightarrow{ON}$ (d) $\overrightarrow{MN}$.



Not drawn accurately

Hint AN is $\frac{1}{4}$ of AB , so $\overrightarrow{AN} = \frac{1}{4}\overrightarrow{AB}$. Then go from O to A to N .

B3 $ABCD$ is a [4 marks]

parallelogram with $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AD} = \mathbf{q}$. M is the midpoint of CD and N is the point on AC with $AN : NC = 1 : 3$. Find, in terms of $\mathbf{p}$ and $\mathbf{q}$, (a) $\overrightarrow{AC}$ (b) $\overrightarrow{AM}$ (c) $\overrightarrow{NM}$.

Answer (a) (b) (c)

c Practice

A-Level

- C1** State, with a reason, whether each vector is parallel to $2\mathbf{a} - 5\mathbf{b}$. (i) [4 marks]
 $6\mathbf{a} - 15\mathbf{b}$ (ii) $5\mathbf{b} - 2\mathbf{a}$ (iii) $4\mathbf{a} - 5\mathbf{b}$ (iv) $-\mathbf{a} + \frac{5}{2}\mathbf{b}$

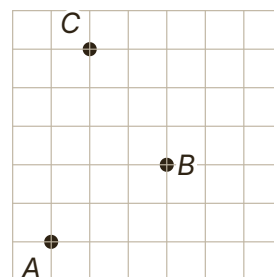
Answer (i) ... (ii) ... (iii) ... (iv) ...

C2 The vectors $3\mathbf{a} + k\mathbf{b}$ and $4\mathbf{a} - 6\mathbf{b}$ are parallel. Find k .

[2 marks]

Answer $k = \dots\dots$

C3 The points A , B and C are shown on a grid of unit squares. (a) Write $\overrightarrow{AB}$ and $\overrightarrow{BC}$ as column vectors. (b) Find $\overrightarrow{AB} + 2\overrightarrow{BC}$ in $\mathbf{i}$, $\mathbf{j}$ form. [4 marks]



Answer (a) $\dots\dots$ (b) $\dots\dots$

D Exam-style questions

Edexcel exam style

D1 Lily says: "The vectors $6\mathbf{i} - 8\mathbf{j}$ and $-9\mathbf{i} + 12\mathbf{j}$ are not parallel, because their $\mathbf{i}$ parts have different signs."

Lily is wrong.

(a) Show that the two vectors are parallel, and say whether they point the same way or opposite ways. [3 marks]

Answer $\dots\dots$

- (b) The vector $(p + 2)\mathbf{i} + 3\mathbf{j}$ is also parallel to $6\mathbf{i} - 8\mathbf{j}$. Find the value of p . [3 marks]

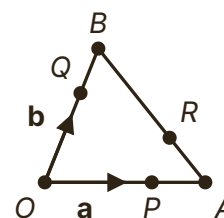
Answer $p = \dots\dots$

Total for D1: 6 marks

- D2** OAB is a triangle with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. P is the point on OA with $OP : PA = 2 : 1$, Q is the point on OB with $OQ : QB = 2 : 1$, and R is the point on AB with $AR : RB = 1 : 2$.

- (a) Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

[1 mark]



Answer $\overrightarrow{AB} = \dots\dots$

Not drawn accurately

- (b) Find $\overrightarrow{PQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

[2 marks]

Answer $\overrightarrow{PQ} = \dots\dots$

- (c) Hence show that PQ is parallel to AB , and find the ratio $PQ : AB$.

[3 marks]

Answer $PQ : AB = \dots\dots$

(d) Show that PR is parallel to OB .

[3 marks]

Total for D2: 9 marks

D3 $\mathbf{a} = 7\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = p\mathbf{i} + p\mathbf{j}$, where p is a constant. The resultant of $\mathbf{a}$ and $\mathbf{b}$ is parallel to $2\mathbf{i} + \mathbf{j}$.

(a) Find the value of p .

[3 marks]

Answer $p = \dots\dots$

(b) Hence find the resultant of $\mathbf{a}$ and $\mathbf{b}$.

[1 mark]

Answer $\dots\dots$

(c) Find the value of μ for which $\mathbf{a} - \mu\mathbf{b}$ is parallel to $\mathbf{j}$.

[3 marks]

Answer $\mu = \dots\dots$

(d) Given that $\lambda \mathbf{a} + \nu \mathbf{b} = 9\mathbf{i} - 3\mathbf{j}$, find λ and ν .

[4 marks]

Answer $\lambda = \dots\dots$ $\nu = \dots\dots$

Total for D3: 11 marks

D4 $OABC$ is a quadrilateral with $\overrightarrow{OA} = 3\mathbf{a}$, $\overrightarrow{OC} = 2\mathbf{c}$ and $\overrightarrow{OB} = 3\mathbf{a} + 4\mathbf{c}$, where $\mathbf{a}$ and $\mathbf{c}$ are not parallel.

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{CB}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

[2 marks]

Answer $\overrightarrow{AB} = \dots\dots$ $\overrightarrow{CB} = \dots\dots$

(b) Hence show that AB is parallel to OC , find the ratio $AB : OC$, and state, with a reason, the type of quadrilateral $OABC$ is.

[4 marks]

Answer $AB : OC = \dots\dots$ type: $\dots\dots$

(c) M is the midpoint of AB . Show that $OAMC$ is a parallelogram. [3 marks]

Total for D4: 9 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $\mathbf{a}$ and $\mathbf{b}$ are not parallel. Find every value of k for which $(k + 1)\mathbf{a} + 4\mathbf{b}$ is parallel to $\mathbf{a} + (k - 2)\mathbf{b}$. For each value, say whether the two vectors point the same way or opposite ways. [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Parallel: $(k + 1)\mathbf{a} + 4\mathbf{b} = \lambda(\mathbf{a} + (k - 2)\mathbf{b})$.
- Hint 2** Compare the $\mathbf{a}$ parts and the $\mathbf{b}$ parts:
 $k + 1 = \lambda$ and $4 = \lambda(k - 2)$.
- Hint 3** So $(k + 1)(k - 2) = 4$. The sign of λ gives the direction.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can add vectors with the triangle law. ☐ ☐ ☐

I can find vectors in a triangle or parallelogram. ☐ ☐ ☐

I can use $\mathbf{i}, \mathbf{j}$ and column vectors. ☐ ☐ ☐

I can show vectors are parallel and find an unknown. ☐ ☐ ☐