Vectors in mechanics

118 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-vectors-in-mechanics/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Find the unit vector in the direction of $2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$.

Answer .....

A2 From Year 1 [2 marks]

A particle of mass 2 kg is acted on by a single force $(6\mathbf{i} - 8\mathbf{j})$ N. Find its acceleration vector and the magnitude of its acceleration.

Answer .....

A3 Last lesson [2 marks]

Show that the points $A(1, 4, -2)$, $B(4, 3, 0)$ and $C(10, 1, 4)$ are collinear.

Answer .....

A4 Common slip [2 marks]

A particle has velocity $(3\mathbf{i} - 4\mathbf{j} + 12\mathbf{k})$ m s^{-1}. Find its speed.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Three forces $(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$ N, $(\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ N and $(3\mathbf{i} + 3\mathbf{j} + \mathbf{k})$ N act on a particle. Find the resultant force and its magnitude.

Hint Add the $\mathbf{i}$ parts, then the $\mathbf{j}$ parts, then the $\mathbf{k}$ parts. The magnitude is found from the resultant, not by adding the three magnitudes.

B1 Three forces $(3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$ N, $(-\mathbf{i} + 5\mathbf{j} + 2\mathbf{k})$ N and $(4\mathbf{i} + \mathbf{j} - 3\mathbf{k})$ N act on a particle. Find the magnitude of the resultant force and the angle it makes with $\mathbf{k}$. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 2

The forces $(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ N, $(3\mathbf{i} + b\mathbf{j} + 4\mathbf{k})$ N and $(-5\mathbf{i} + \mathbf{j} + c\mathbf{k})$ N act on a particle in equilibrium. Find a, b and c.

Hint In equilibrium the resultant is the zero vector, so the $\mathbf{i}$ parts add to 0, the $\mathbf{j}$ parts add to 0 and the $\mathbf{k}$ parts add to 0.

B2 [4 marks]

The forces $\mathbf{F}_1 = (2\mathbf{i} + p\mathbf{j} - 3\mathbf{k})$ N, $\mathbf{F}_2 = (-5\mathbf{i} + 2\mathbf{j} + q\mathbf{k})$ N, $\mathbf{F}_3 = (r\mathbf{i} - 6\mathbf{j} + 4\mathbf{k})$ N and $\mathbf{F}_4 = (\mathbf{i} + \mathbf{j} + \mathbf{k})$ N act on a particle in equilibrium. Find p, q and r. The force $\mathbf{F}_4$ is then removed. Find the magnitude of the resultant of the other three.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 3

A particle of mass 2 kg is acted on by the forces $(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ N and $(5\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$ N only. Find its acceleration and the magnitude of the acceleration.

Hint $\mathbf{F} = m\mathbf{a}$ works on the whole vector: find the resultant force, then divide every component by the mass.

B3 A particle of mass 3 kg is acted on by the forces $(4\mathbf{i} - 5\mathbf{j} + 7\mathbf{k})$ N and $(2\mathbf{i} + 11\mathbf{j} - \mathbf{k})$ N only. Find its acceleration, the magnitude of the acceleration, and the angle the acceleration makes with $\mathbf{j}$. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 4

A drone starts at the point with position vector $(2\mathbf{i} - \mathbf{j} + 4\mathbf{k})$ m and moves with constant velocity $(3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ m s^{-1}. Find its position after 4 s and its distance from O then.

Hint At constant velocity, $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$.

B4 A drone starts at $(5\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ m and moves with constant velocity $(-2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$ m s^{-1}. $\mathbf{k}$ is vertically up. Find its position after 2 s, the time at which it is 13 m above the ground, and the distance it has travelled by then. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 5

A particle has initial velocity $(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ m s^{-1} and constant acceleration $(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ m s^{-2}. Find its velocity and speed after 3 s.

Hint The constant acceleration formulae hold with vectors: $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ and $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$.

B5 A particle has initial velocity $(4\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ m s^{-1} and constant acceleration $(-2\mathbf{i} + 3\mathbf{j} + \mathbf{k})$ m s^{-2}. Find the time at which it moves parallel to $\mathbf{j}$, its speed then, and its displacement from its start at that time. [6 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C1 Two forces $(2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k})$ N and $(4\mathbf{i} - 4\mathbf{j} - 7\mathbf{k})$ N act on a particle. Leo says the magnitudes are 7 N and 9 N, so the resultant has magnitude 16 N. Explain his mistake and find the magnitude of the resultant. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C2 A particle of mass 2 kg moves under gravity and a force $(4\mathbf{i} - 2\mathbf{j} + 30\mathbf{k})$ N only, where $\mathbf{k}$ is vertically up. Take $g = 9.8$ m s^{-2}. Sam writes $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 15\mathbf{k}$. Find his mistake and the magnitude of the acceleration. [5 marks]

Answer .....

C3 A particle is in equilibrium under three forces. $\mathbf{F}_1 = (3\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ N and $\mathbf{F}_2 = (-\mathbf{i} + 5\mathbf{j} + 2\mathbf{k})$ N. Ana writes $\mathbf{F}_3 = \mathbf{F}_1 + \mathbf{F}_2 = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$. Correct her answer and find the exact magnitude of $\mathbf{F}_3$. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C4 The force $\mathbf{F} = (p\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$ N has magnitude 9 N, and $p > 0$. Find p and the angle $\mathbf{F}$ makes with $\mathbf{k}$. Explain why the answer is not $48.2°$. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C5 A boat moves with constant velocity. At $t = 0$ it is at $A(2, -3, 5)$ and at $t = 4$ s it is at $B(14, 1, -1)$, in metres. Find its velocity, its speed, and its position at $t = 6$ s. [5 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 Two particles P and Q move with constant velocities. At $t = 0$, P is at $(\mathbf{i} + 7\mathbf{j} + 2\mathbf{k})$ m and Q is at $(13\mathbf{i} + \mathbf{j} + 8\mathbf{k})$ m. P moves with velocity $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ m s^{-1} and Q with velocity $(-\mathbf{i} + \mathbf{j})$ m s^{-1}.

(a) Write down the position vectors of P and Q at time t seconds. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Show that the particles collide, and find the position vector of the point C where they collide. [4 marks]

Answer .....

(c) Find the distance between the particles at $t = 1$. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(d) Find the exact distance of C from O. [2 marks]

Answer .....

Total for D1: 10 marks

D2

A particle of mass 2 kg moves under a single constant force. Its velocity is $(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ m s^{-1} at $t = 0$ and $(9\mathbf{i} + 6\mathbf{j} - 6\mathbf{k})$ m s^{-1} at $t = 4$ s. Jo finds the speeds, 3 and $\sqrt{153}$, and says the force has magnitude $2 \times \dfrac{\sqrt{153} - 3}{4} \approx 4.7$ N.

Jo is wrong.

(a) Find the acceleration of the particle. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Find the magnitude of the force. [2 marks]

Answer .....

(c) Explain Jo's mistake. [2 marks]

Answer .....

(d) Find the speed of the particle at $t = 6$ s. [2 marks]

Answer .....

Total for D2: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D3 A particle of mass 4 kg is released from rest at O. It moves under gravity and two constant forces $(6\mathbf{i} - 3\mathbf{j} + 40\mathbf{k})$ N and $(2\mathbf{i} + 7\mathbf{j} + 7.2\mathbf{k})$ N, where $\mathbf{k}$ is vertically up and $g = 9.8$ m s^{-2}. Find the distance of the particle from O after 5 s, and its speed at that moment. [7 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D4 A particle has initial velocity $(-5\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$ m s^{-1} and constant acceleration $(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ m s^{-2}.

(a) Find the velocity of the particle at time t seconds. [2 marks]

Answer .....

(b) Show that the square of the speed at time t is $9t^2 - 36t + 38$. [3 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Hence find the least speed of the particle and the time at which it happens. [3 marks]

Answer .....

(d) Find the times at which the speed is $\sqrt{11}$ m s^{-1}. [2 marks]

Answer .....

Total for D4: 10 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D5 A particle of mass m kg hangs in equilibrium under gravity and the forces $\mathbf{F}_1 = (4\mathbf{i} - 2\mathbf{j} + 20\mathbf{k})$ N, $\mathbf{F}_2 = (-\mathbf{i} + 6\mathbf{j} + 15\mathbf{k})$ N and $\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j} + 14\mathbf{k})$ N. $\mathbf{k}$ is vertically up and $g = 9.8$ m s^{-2}.

(a) Find p and q. [2 marks]

Answer .....

(b) Find m. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) The force $\mathbf{F}_3$ is removed. Find the acceleration of the particle and its magnitude. [3 marks]

Answer .....

(d) Find the angle between this acceleration and the downward vertical. [2 marks]

Answer .....

Total for D5: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D6 A ball is projected from O on horizontal ground with velocity $(6\mathbf{i} + 8\mathbf{j} + 19.6\mathbf{k})$ m s^{-1}, where $\mathbf{k}$ is vertically up. It is modelled as a particle moving under gravity only, with $g = 9.8$ m s^{-2}.

(a) Find the position vector of the ball at time t seconds. [2 marks]

Answer .....

(b) Find the time the ball lands and its distance from O when it lands. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(c) Find the greatest height of the ball, and state one way in which the model could be improved. [3 marks]

Answer .....

Total for D6: 9 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [9 marks]

Two drones A and B move with constant velocities. At time t seconds their position vectors, in metres, are $\mathbf{a} = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + t(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ and $\mathbf{b} = (7\mathbf{i} + 9\mathbf{k}) + t(-\mathbf{i} + \mathbf{j} - \mathbf{k})$. Show that the drones do not collide. Find the least distance between them and where each drone is at that moment.

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ in terms of t.

Hint 2 The square of the distance is a quadratic in t. Complete the square.

Hint 3 If the least value of the square of the distance is not zero, they never meet.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the resultant of several forces and the unknown forces that keep a particle in equilibrium.	○ ○ ○
I can use F = ma with force and acceleration vectors, including the weight of the particle.	○ ○ ○
I can use r = r0 + vt and the constant acceleration formulae in vector form.	○ ○ ○
I can find a speed, an angle with an axis, and the least distance between two moving particles.	○ ○ ○

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.