

Where a line meets a circle

75 marks



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mathswariz.uk/worksheets/a-level-where-a-line-meets-a-circle/

Date

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] Find the discriminant of $3x^2 + 5x + 4$. How many real roots does $3x^2 + 5x + 4 = 0$ have?
- A2** Needed today [2 marks] Substitute $y = 3x - 2$ into $x^2 + y^2 = 10$. Write the result in the form $ax^2 + bx + c = 0$.

Answer

Answer

- A3** Last lesson [2 marks] Find the centre and the radius of the circle $x^2 + y^2 - 8x + 6y - 11 = 0$.
- A4** From chapter 5 [2 marks] Find the equation of the line through $(2, -1)$ perpendicular to $y = 3x + 4$, in the form $ax + by + c = 0$.

Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

The circle $(x - 3)^2 + (y + 2)^2 = 20$ meets the x -axis at two points. Find their coordinates.

Hint On the x -axis, $y = 0$. Substitute and solve for x .

B1 The circle [3 marks]

$(x + 2)^2 + (y - 4)^2 = 25$ meets the y -axis at two points. Find their coordinates, giving your answers in surd form.

Answer

EXAMPLE 2

Find the coordinates of the points where the line $y = x - 1$ meets the circle $(x - 2)^2 + (y - 3)^2 = 10$.

Hint Replace y by $x - 1$ in the circle's equation. You get a quadratic in x . Then find each y from the LINE.

B2 The line $x + y = 4$ [4 marks]

meets the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ at A and B . Find the coordinates of A and B .

Answer (.....,.....) and (.....,.....)

EXAMPLE 3

Show that the line $y = 2x + 6$ does not meet the circle

$$(x - 4)^2 + (y - 1)^2 = 9.$$

Hint Substitute, simplify to

$ax^2 + bx + c = 0$, then look at the sign of $b^2 - 4ac$.

B3 Show that the line [4 marks]

$3x + y = 15$ does not meet the circle

$$x^2 + y^2 + 2x - 4y - 4 = 0.$$

EXAMPLE 4

The line $y = x + k$ is a tangent to the circle $(x - 3)^2 + y^2 = 8$. Find the two possible values of k and the point of contact for each.

Hint A tangent meets the circle at ONE

point, so the quadratic has a repeated root: $b^2 - 4ac = 0$.

B4 The line $y = 2x + c$ is a [5 marks]

tangent to the circle

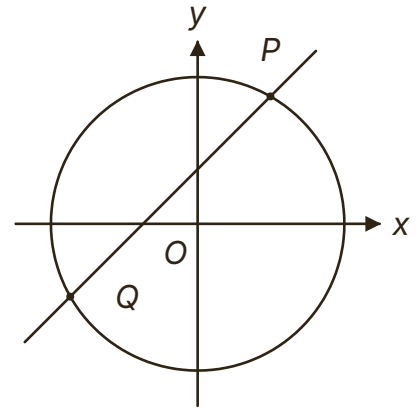
$$(x - 1)^2 + (y - 2)^2 = 20.$$

Find the two possible values of c .

Answer $c = \dots\dots$ or $c = \dots\dots$

- C1** The line $x - 2y + 5 = 0$ meets the circle $(x + 1)^2 + (y - 2)^2 = 45$ at A and B . Show that AB is a diameter of the circle. [4 marks]

- C2** The line $x - y + 3 = 0$ meets the circle $x^2 + y^2 = 65$ at P and Q . Find the exact length of PQ . [5 marks]



Not drawn accurately

Answer $PQ = \dots\dots$

- C3** Show that the line $x + 3y = 9$ is a tangent to the circle $(x - 2)^2 + (y + 1)^2 = 10$, and find the coordinates of the point where it touches the circle. [4 marks]

Answer $(\dots\dots, \dots\dots)$

D1

Priya is asked whether the line $y = 9 - 2x$ meets the circle $x^2 + y^2 - 2x - 4y - 5 = 0$. She writes:

$$x^2 + 81 + 4x^2 - 2x - 36 + 8x - 5 = 0, \text{ so } 5x^2 + 6x + 40 = 0.$$

$$b^2 - 4ac = 36 - 800 < 0, \text{ so the line does not meet the circle.}$$

Priya is wrong.

(a) Identify the mistake Priya has made. [1 mark]

(b) Find the coordinates of the points A and B where the line meets the circle. [4 marks]

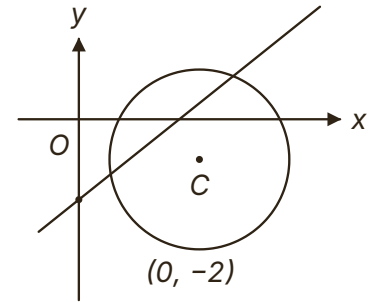
Answer (.....,.....) and (.....,.....)

(c) Show that the perpendicular bisector of AB passes through the centre of the circle. [3 marks]

Total for D1: 8 marks

D2 The circle C has equation $x^2 + y^2 - 6x + 2y + 5 = 0$. The line l has equation $y = kx - 2$, where k is a constant.

- (a) Show that the x -coordinates of any points where l meets C satisfy $(1 + k^2)x^2 - (2k + 6)x + 5 = 0$. [2 marks]



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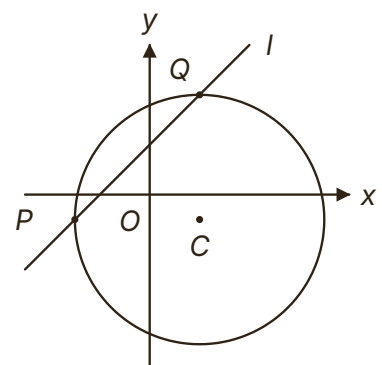
- (b) Given that l meets C at two distinct points, show that $2k^2 - 3k - 2 < 0$. Hence find the range of possible values of k . [5 marks]

Answer

Total for D2: 7 marks

D3 The circle with equation $(x - 2)^2 + (y + 1)^2 = 25$ has centre C . The line l has equation $y = x + 2$. The line l meets the circle at P and Q , where P is to the left of Q .

- (a) Find the coordinates of P and Q . [5 marks]



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Answer $P(\dots, \dots)$, $Q(\dots, \dots)$

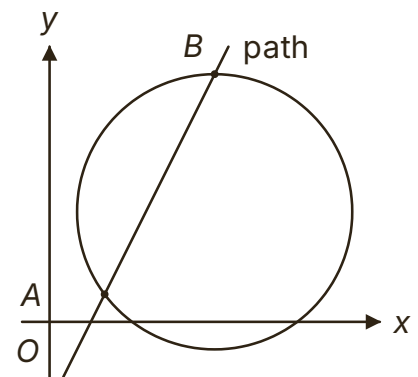
- (b) Show that angle PCQ is 90° . Hence find the exact area of the smaller region bounded by the circle and the chord PQ . [6 marks]

Answer

Total for D3: 11 marks

- D4** A circular lawn is modelled by the circle $x^2 + y^2 - 12x - 8y + 27 = 0$, where the units are metres. A straight path is modelled by the line $y = 2x - 3$. The path crosses the edge of the lawn at A and B . [6 marks]

Find the length AB of the path that crosses the lawn, giving your answer in the form $a\sqrt{5}$ metres.



Not drawn accurately

Answer m

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The line $y = kx$ meets the circle $x^2 + y^2 - 10x + 16 = 0$ at two distinct points A and B . Find the range of values of k . Show that, as k varies, the midpoint of AB always lies on one fixed circle, and find the centre and radius of that circle. [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write down the quadratic in x . The midpoint's x -coordinate is half the SUM of the roots, which you can read off the quadratic without solving it.
- Hint 2** $(1 + k^2)x^2 - 10x + 16 = 0$. The midpoint is $\left(\frac{5}{1 + k^2}, \frac{5k}{1 + k^2}\right)$.
- Hint 3** $100 - 64(1 + k^2) > 0$ gives $-\frac{3}{4} < k < \frac{3}{4}$. The midpoint has $x^2 + y^2 = \frac{25}{1 + k^2} = 5x$: the circle with centre $\left(\frac{5}{2}, 0\right)$ and radius $\frac{5}{2}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can find where a line meets a circle by substituting and solving a quadratic. ○ ○ ○
- I can use the discriminant to decide if a line crosses, touches or misses a circle. ○ ○ ○
- I can find an unknown constant when a line is a tangent or misses a circle. ○ ○ ○
- I can use the points found to work out lengths, midpoints and areas. ○ ○ ○