



Algebraic fractions, rearranging formulae, composite and inverse functions, and iteration

Grades 4–9 About 85 minutes

Calculator allowed 92 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 Factorise $x^2 - 9$.

[1 mark]

Answer

A2 Factorise $x^2 + 9x + 14$.

[1 mark]

Answer

A3 Work out $\frac{3}{4} + \frac{1}{6}$.

[1 mark]

Answer

A4 Make x the subject of $y = 5x - 2$.

[2 marks]

Answer

A5 Solve $x^2 - 2x - 15 = 0$.

[2 marks]

Answer

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

Simplify fully $\frac{x^2 + 3x - 10}{x^2 - 4}$.

Hint Factorise the top and the bottom. Then cancel the bracket they share.

B1 Simplify fully $\frac{x^2 - 2x - 8}{x^2 - 16}$. [3 marks]

Answer

EXAMPLE 2

Write $\frac{3}{x+1} + \frac{2}{x-2}$ as a single fraction.

Hint Use $(x+1)(x-2)$ as the common denominator.

B2 Write $\frac{4}{x-3} - \frac{1}{x+2}$ as a single fraction in its simplest form. [3 marks]

Answer

EXAMPLE 3

Make y the subject of $a(y+1) = 2y - 5$.

Hint Expand the bracket. Collect every term with y on one side, then take y out as a factor.

B3 Make y the subject of $t(y-2) = 3y + 1$. [3 marks]

Answer

EXAMPLE 4

$f(x) = 2x + 3$ and $g(x) = x^2$. Find $fg(2)$ and $f^{-1}(x)$.

Hint $fg(2)$ means do g first. For the inverse, write $y = 2x + 3$ and make x the subject.

B4 $f(x) = 5x - 1$ and $g(x) = x^2 + 2$. Find $gf(1)$ and $f^{-1}(x)$. [3 marks]

Answer $gf(1) = \dots\dots$ $f^{-1}(x) = \dots\dots$

c Practice

Grades 5–7

Each question asks for something different. Show your working.

C1 Simplify fully $\frac{2x^2 + 7x + 3}{4x^2 - 1}$. [3 marks]

Answer

C2 Simplify fully $\frac{x^2 - 25}{x + 2} \div \frac{x + 5}{2x^2 + 4x}$. [3 marks]

Answer

C3 Solve $\frac{3}{x - 1} + \frac{2}{x + 2} = 1$. Give your answers to 2 decimal places. [4 marks]

Answer

C4 Make x the subject of $y = \frac{x+4}{x-3}$.

[3 marks]

Answer

C5 $f(x) = 3x - 1$ and $g(x) = \frac{1}{x+2}$.

(a) Find $gf(3)$.

[2 marks]

Answer

(b) Find $fg(x)$. Give your answer as a single fraction.

[2 marks]

Answer

(c) Which value of x cannot be used as an input to g ?

[1 mark]

Answer

Total for C5: 5 marks

C6 $f(x) = \frac{2x+7}{3}$. Find $f^{-1}(x)$.

[2 marks]

Answer

C7 Use the iteration formula $x_{n+1} = \sqrt[3]{5x_n + 3}$ with $x_0 = 2$.

(a) Work out x_1 . Give your answer to 4 decimal places.

[1 mark]

Answer

(b) Work out x_3 . Give your answer to 4 decimal places.

[2 marks]

Answer

Total for C7: 3 marks

C8 Show that the equation $x^2 - 7x + 2 = 0$ can be rearranged to give $x = 7 - \frac{2}{x}$. [2 marks]

D Exam-style questions

Grades 6–9

Answer every question in the space given. Show your working: most of the marks are for method.

D1 Show that $\frac{3}{x+2} - \frac{2}{x-1}$ can be written as $\frac{x-7}{(x+2)(x-1)}$. [3 marks]

D2 Ben says, " $\frac{x+6}{x+3}$ simplifies to 2, because $6 \div 3 = 2$." [2 marks]

Ben is wrong. Explain why.

Answer

D3 $f(x) = x + 3$ and $g(x) = 2x^2$.

[4 marks]

Solve $fg(x) = gf(x)$. You must show your working.

Answer

D4 $f(x) = 4 - 3x$.

[3 marks]

Solve $f(x) = f^{-1}(x)$.

Answer

D5 The equation $x^3 + 4x - 9 = 0$ has one solution.

(a) Show that the solution lies between 1 and 2.

[2 marks]

(b) Show that the equation can be rearranged to give $x = \sqrt[3]{9 - 4x}$.

[1 mark]

(c) Use the iteration formula $x_{n+1} = \sqrt[3]{9 - 4x_n}$ with $x_0 = 1.5$ to find x_1 , x_2 and x_3 . Give each answer to 4 decimal places.

[3 marks]

Answer $x_1 = \dots\dots$ $x_2 = \dots\dots$ $x_3 = \dots\dots$

Total for D5: 6 marks

D6 Make t the subject of the formula $s = \frac{a + bt}{1 - t}$. [4 marks]

Answer

D7 $f(x) = x^2 + 6x + 4$ for $x \geq -3$. [4 marks]
Find $f^{-1}(x)$.

Answer

D8 This question is about the sum $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5}$.
(a) Show that $\frac{1}{n} - \frac{1}{n+1} = \frac{1}{n(n+1)}$. [2 marks]

(b) Hence work out the value of the sum. [1 mark]

Answer

Total for D8: 3 marks

D9 A sequence is given by $x_{n+1} = 2 + \frac{3}{x_n}$ with $x_1 = 1$.
(a) Work out x_2 , x_3 and x_4 . Give x_4 to 2 decimal places. [2 marks]

Answer $x_2 = \dots\dots$ $x_3 = \dots\dots$ $x_4 = \dots\dots$

- (b) The values get closer and closer to a number L . By solving $L = 2 + \frac{3}{L}$, find L . [2 marks]

Answer

Total for D9: 4 marks

- D10** $f(x) = 3x + k$, where k is a constant. $ff(2) = 32$. [3 marks]

Find the value of k .

Answer

E Extension

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** $f(x) = \frac{x+5}{x-1}$, $x \neq 1$. [3 marks]

Show that f is its own inverse.

- E2** Simplify fully $\frac{x^4 - 16}{x^3 + 2x^2 + 4x + 8}$. [3 marks]

Answer

E3

Make x the subject of $y = \sqrt{\frac{x+a}{x-a}}$.

[3 marks]

Answer

E4

$f(x) = x^2 - 2x$ and $g(x) = 2x + 1$.

[3 marks]

Solve $fg(x) = gf(x)$. Give your answers in surd form.

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can simplify, add, subtract, multiply and divide algebraic fractions.	☐	☐	☐
I can make a letter the subject of a formula when it appears more than once.	☐	☐	☐
I can use function notation, work out composite functions and find an inverse function.	☐	☐	☐
I can show that an equation has a solution in an interval and use an iteration formula to find it.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

E1	Hint 1	Find $f^{-1}(x)$ in the usual way and compare it with $f(x)$.
	Hint 2	Write $y(x - 1) = x + 5$, so $xy - y = x + 5$.
	Hint 3	$x(y - 1) = y + 5$, so $x = \frac{y + 5}{y - 1}$. That gives $f^{-1}(x) = \frac{x + 5}{x - 1}$, which is $f(x)$.

E2	Hint 1	Factorise the top as a difference of two squares, twice if you can.
	Hint 2	$x^4 - 16 = (x^2 + 4)(x + 2)(x - 2)$. Factorise the bottom in pairs.
	Hint 3	$x^3 + 2x^2 + 4x + 8 = x^2(x + 2) + 4(x + 2) = (x + 2)(x^2 + 4)$, so the fraction is $x - 2$.

E3	Hint 1	Square both sides first.
	Hint 2	$y^2(x - a) = x + a$, so $xy^2 - ay^2 = x + a$.
	Hint 3	$x(y^2 - 1) = a(y^2 + 1)$, so $x = \frac{a(y^2 + 1)}{y^2 - 1}$.

E4	Hint 1	Work out each composite function and set them equal.
	Hint 2	$fg(x) = (2x + 1)^2 - 2(2x + 1) = 4x^2 - 1$ and $gf(x) = 2x^2 - 4x + 1$.
	Hint 3	$2x^2 + 4x - 2 = 0$, so $x^2 + 2x - 1 = 0$ and $x = -1 \pm \sqrt{2}$.