



The addition rule, tree diagrams and conditional probability

Grades 4–9 About 80 minutes No calculator 90 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 Work out $\frac{3}{5} \times \frac{2}{7}$.

[1 mark]

Answer

A2 The probability that it rains tomorrow is 0.35. What is the probability that it does not rain tomorrow? [1 mark]

Answer

A3 Work out $\frac{1}{4} + \frac{1}{6}$.

[1 mark]

Answer

A4 A bag holds 4 red counters and 6 blue counters. One counter is taken at random. Write down the probability that it is red. Give your answer in its simplest form. [1 mark]

Answer

A5 A coin is thrown and a four-sided spinner numbered 1 to 4 is spun. How many different outcomes are there? [1 mark]

Answer

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

A spinner can land on red, blue or green. The probability of red is 0.3 and the probability of blue is 0.45. Work out the probability that it lands on red or blue.

Hint The spinner cannot land on red and blue at the same time, so the two probabilities can be added.

- B1** Events A and B cannot both happen. $P(A) = 0.28$ and $P(B) = 0.35$. Work out the probability that neither A nor B happens. [2 marks]

Answer

EXAMPLE 2

$P(A) = 0.6$, $P(B) = 0.5$ and $P(A \text{ and } B) = 0.3$. Work out $P(A \text{ or } B)$.

Hint Adding $P(A)$ and $P(B)$ counts the overlap twice, so take it off once.

- B2** $P(A) = 0.45$, $P(B) = 0.4$ and $P(A \text{ or } B) = 0.65$. Work out $P(A \text{ and } B)$. [2 marks]

Answer

EXAMPLE 3

The probability that Kai wins a game is 0.7. He plays two games, and the results are independent. Work out the probability that he wins exactly one of them.

Hint Win then lose, or lose then win. Multiply along each branch, then add the two branches.

- B3** The probability that a bus is late is 0.2. Lena catches the bus on two days, and the days are independent. Work out the probability that the bus is late on exactly one of the two days. [3 marks]

Answer

EXAMPLE 4

A bag holds 5 red sweets and 3 green sweets. Two sweets are taken at random without replacement. Work out the probability that both are red.

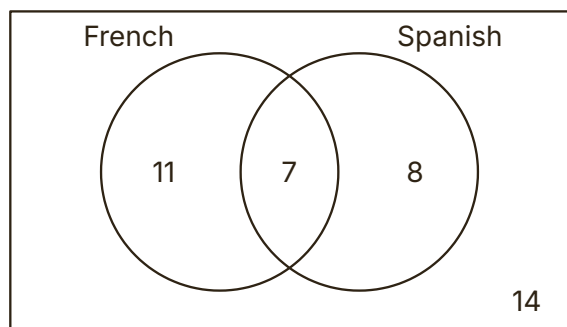
Hint After one red sweet is taken there are 4 red sweets left out of 7.

- B4** A bag holds 4 blue beads and 6 yellow beads. Two beads are taken at random without replacement. Work out the probability that they are the same colour. [3 marks]

Answer

Each question asks for something different. Show your working.

- C1** The Venn diagram shows how many of 40 students study French and how many study Spanish. One of the 40 students is chosen at random.



- (a) Write down the probability that the student studies both languages. [1 mark]

Answer

- (b) Work out the probability that the student studies French or Spanish. [2 marks]

Answer

- (c) The student chosen studies French. Work out the probability that this student also studies Spanish. [2 marks]

Answer

Total for C1: 5 marks

- C2** The two-way table shows whether 120 adults drive to work.

	Drives	Does not drive
Men	34	26
Women	38	22

- (a) One adult is chosen at random. Work out the probability that this adult is a woman who drives. [2 marks]

Answer

- (b) One of the women is chosen at random. Work out the probability that she drives. [2 marks]

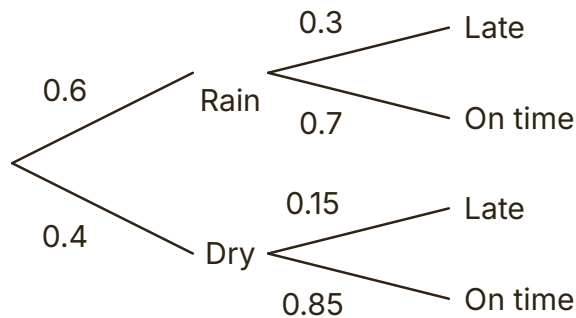
Answer

Total for C2: 4 marks

- C3 A and B are independent events. $P(A) = 0.4$ and $P(B) = 0.25$. Work out $P(A \text{ or } B)$. [3 marks]

Answer

- C4 The tree diagram shows the probability that it rains on a school day, and the probability that Sophie is late for school. Work out the probability that Sophie is late. [3 marks]



Answer

- C5 A four-sided spinner numbered 1 to 4 is spun and a fair dice numbered 1 to 6 is thrown. The two numbers are added together.

- (a) Work out the probability that the total is 7. [2 marks]

Answer

- (b) The two numbers are multiplied instead. Work out the probability that the product is even. [2 marks]

Answer

Total for C5: 4 marks

- C6 A fair dice is thrown twice. Work out the probability of getting at least one 6. [2 marks]

Answer

- C7 $P(A) = 0.5$. The probability that B happens, given that A happens, is 0.3. Work out the probability that both A and B happen. [2 marks]

Answer

- C8 A bag holds 3 red counters and 7 blue counters. A counter is taken at random, its colour is written down and it is put back. A second counter is then taken. Work out the probability that the two counters are different colours. [3 marks]

Answer

- C9 A coin is biased so that the probability of a head is 0.6. The coin is thrown three times. Work out the probability of exactly two heads. [3 marks]

Answer

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** 50 people were asked whether they like tea and whether they like coffee. 28 like tea, 30 like coffee and 6 like neither. One of the 50 people is chosen at random. Work out the probability that this person likes both tea and coffee. [3 marks]

Answer

- D2** Mia says, "When two fair dice are thrown, the probability of two sixes is $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$." [2 marks]
Mia is wrong. Explain why, and work out the correct probability.

Answer

- D3** A bag holds n counters. 5 of the counters are red and the rest are blue. Two counters are taken at random without replacement. The probability that both counters are red is $\frac{2}{9}$. [3 marks]
- (a) Show that $n^2 - n - 90 = 0$.

- (b) Work out the value of n . [1 mark]

Answer $n = \dots\dots$

- (c) Work out the probability that the two counters taken are different colours. [2 marks]

Answer

Total for D3: 6 marks

- D4** There are 200 students in a year group. 120 of them study art, 80 study music, and 45 study both. A student who studies music is chosen at random. Work out the probability that this student also studies art. [3 marks]

Answer

- D5** The probability that Jo passes her driving test at the first attempt is 0.6. If she fails, the probability that she passes at the second attempt is 0.7. Work out the probability that she passes within her first two attempts. [3 marks]

Answer

- D6** $P(A) = 0.3$, $P(B) = 0.6$ and $P(A \text{ and } B) = 0.2$. Are A and B independent? You must show your working. [2 marks]

Answer

- D7** Two fair dice are thrown. The score is the difference between the two numbers. Work out the probability that the score is 0 or 1. [3 marks]

Answer

- D8** A car passes three sets of traffic lights. At each set the probability that the lights are red is 0.4, independently of the others. Work out the probability that the car meets at least one red light. [3 marks]

Answer

- D9** $P(A) = 0.45$, $P(B) = 0.3$ and $P(A \text{ or } B) = 0.6$. Work out the probability that B happens, given that A happens. [3 marks]

Answer

- D10** There are 80 people at a meeting. 35 of them are men. 28 of the women and 12 of the men wear glasses. A person who wears glasses is chosen at random. Work out the probability that this person is a man. [3 marks]

Answer

E Extension

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** A bag holds 3 red counters and x blue counters. Two counters are taken at random without replacement. The probability that they are different colours is $\frac{1}{2}$. Work out the possible values of x . [4 marks]

Answer

E2 Ali and Ben take turns to throw a fair dice. Ali goes first. The first person to throw a 6 wins. Work out the probability that Ali wins. [4 marks]

Answer

E3 A factory has two machines. Machine A makes 60% of the items and machine B makes the rest. 2% of the items made by machine A are faulty, and 5% of the items made by machine B are faulty. An item is chosen at random and is found to be faulty. Work out the probability that it was made by machine B. [4 marks]

Answer

E4 A biased dice is thrown twice. The probability of getting two sixes is 0.09. Work out the probability of getting exactly one six. [3 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can add probabilities when two events cannot both happen, and take the overlap off when they can.	☐	☐	☐
I can use a tree diagram for independent events and for events without replacement.	☐	☐	☐
I can work out a conditional probability from a Venn diagram, a two-way table or a description.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

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- E1**
- Hint 1** Different colours means red then blue, or blue then red. Write each in terms of x .
- Hint 2** $2 \times \frac{3}{x+3} \times \frac{x}{x+2} = \frac{1}{2}$
- Hint 3** $12x = (x+3)(x+2)$, so $x^2 - 7x + 6 = 0$, which gives $x = 1$ or $x = 6$. Both work.
-
- E2**
- Hint 1** Ali can win on his first throw, his second throw, his third throw, and so on. Write down the probability of each.
- Hint 2** $\frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \dots$
- Hint 3** Each term is $\frac{25}{36}$ of the one before, so the total is $\frac{1}{6} \div \left(1 - \frac{25}{36}\right) = \frac{6}{11}$.
-
- E3**
- Hint 1** Take 1000 items. Work out how many come from each machine, and how many of those are faulty.
- Hint 2** Machine A makes 600 items and 12 are faulty. Machine B makes 400 items and 20 are faulty.
- Hint 3** 32 items are faulty, and 20 of them come from machine B, so the probability is $\frac{20}{32} = \frac{5}{8}$.
-
- E4**
- Hint 1** Call the probability of a six p . Two sixes is $p \times p$.
- Hint 2** $p^2 = 0.09$, so $p = 0.3$.
- Hint 3** Exactly one six is six then not six, or not six then six: $2 \times 0.3 \times 0.7 = 0.42$.