



Completing the square

Grades 5–9 About 70 minutes No calculator 85 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 Expand $(x + 5)^2$.

[1 mark]

Answer

A2 Expand $(x - 3)^2$.

[1 mark]

Answer

A3 Solve $(x - 2)^2 = 49$. Give both answers.

[2 marks]

Answer

A4 Write $\sqrt{48}$ in the form $a\sqrt{3}$.

[1 mark]

Answer

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

Write $x^2 + 16x + 9$ in the form $(x + p)^2 + q$.

Hint Halve the coefficient of x to find p . Then take away the extra number the bracket makes.

B1 Write $x^2 - 14x + 30$ in the form $(x + p)^2 + q$. [2 marks]

Answer

EXAMPLE 2

Solve $x^2 - 14x + 37 = 0$. Give your answers in the form $a \pm b\sqrt{3}$.

Hint Complete the square first. Then take the square root of both sides, and remember a square root has two answers.

B2 Solve $x^2 + 10x + 13 = 0$. Give your answers in the form $a \pm b\sqrt{3}$. [3 marks]

Answer

EXAMPLE 3

Write $3x^2 + 12x + 7$ in the form $a(x + b)^2 + c$.

Hint Take the 3 out of the first two terms only. Complete the square inside the bracket, then multiply back out.

B3 Write $2x^2 + 12x + 5$ in the form $a(x + b)^2 + c$. [3 marks]

Answer

Each question asks for something different. Show your working.

C1 Write $x^2 + 18x + 85$ in the form $(x + p)^2 + q$. [2 marks]

Answer

C2 Write $x^2 - 11x + 20$ in the form $(x + p)^2 + q$. [2 marks]

Answer

C3 Solve $x^2 - 12x + 31 = 0$ by completing the square. Give your answers in the form $a \pm \sqrt{b}$. [3 marks]

Answer

C4 Find the coordinates of the turning point of the curve $y = x^2 - 10x + 31$. [2 marks]

Answer

C5 Write $3x^2 - 18x + 20$ in the form $a(x + b)^2 + c$.

[3 marks]

Answer

C6 Show that $x^2 - 10x + 34$ is positive for every value of x .

[2 marks]

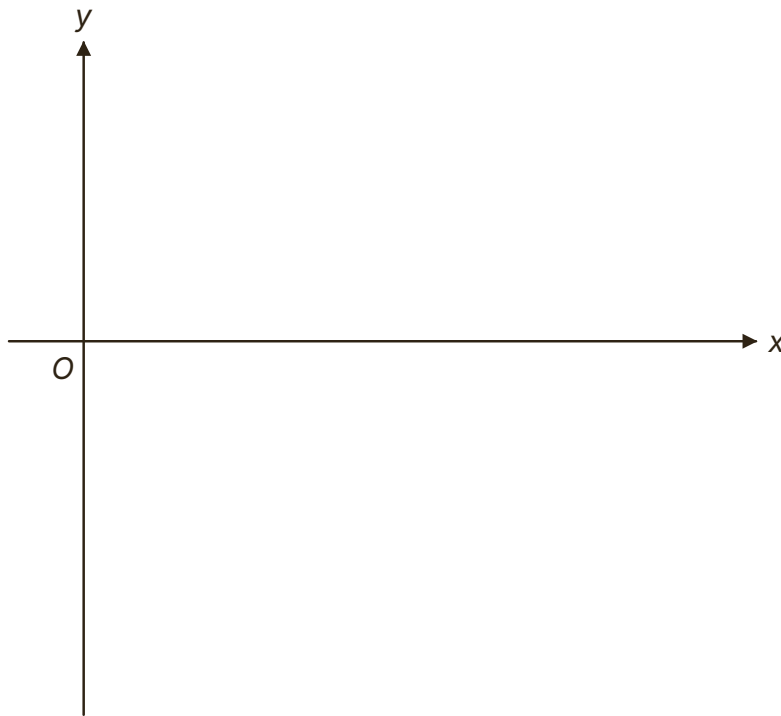
C7 $y = x^2 - 12x + 11$

(a) Write $x^2 - 12x + 11$ in the form $(x + p)^2 + q$.

[2 marks]

Answer

- (b) Sketch the curve on the axes. Mark the coordinates of the turning point and of the points where the curve meets the axes. [2 marks]



Total for C7: 4 marks

D Exam-style questions

Grades 7–9

Answer every question in the space given. Show your working: most of the marks are for method.

D1 $x^2 + 12x + k \equiv (x + a)^2 - 3$ for all values of x . [3 marks]

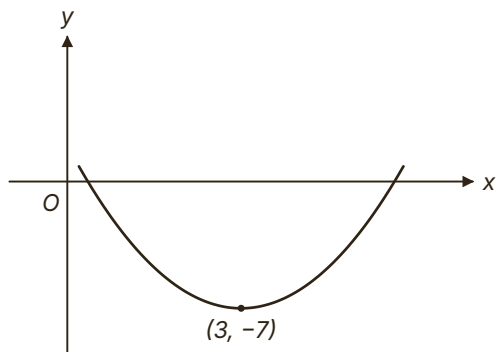
Find the value of a and the value of k .

Answer $a = \dots\dots$ $k = \dots\dots$

- D2** The diagram shows a sketch of the curve $y = x^2 + bx + c$.
The turning point of the curve is $(3, -7)$.

[3 marks]

Find the value of b and the value of c .



Not drawn accurately

Answer $b = \dots\dots$ $c = \dots\dots$

- D3** Sam says, "The minimum value of $x^2 - 14x + 20$ is 20, because the smallest x^2 can be is 0."

[3 marks]

Sam is wrong. Explain why, and find the minimum value of $x^2 - 14x + 20$.

Answer

- D4** Show that the solutions of $x^2 + 12x + 29 = 0$ are $x = -6 \pm \sqrt{7}$.

[3 marks]

D5 Write $2x^2 - 16x + 35$ in the form $a(x + b)^2 + c$.

[4 marks]

Hence write down the coordinates of the turning point of the curve $y = 2x^2 - 16x + 35$.

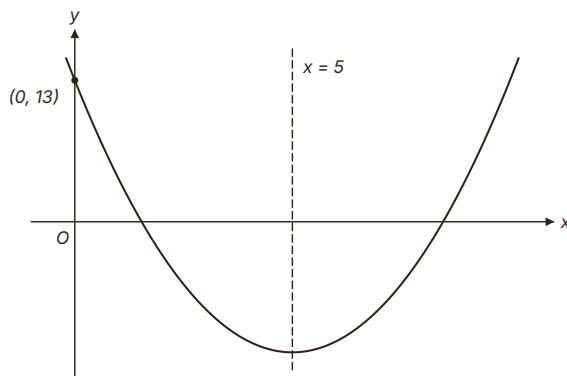
Answer turning point (.....,)

D6 The diagram shows a sketch of the curve $y = x^2 + px + q$.

[4 marks]

The curve meets the y -axis at $(0, 13)$. The line $x = 5$ goes through the turning point.

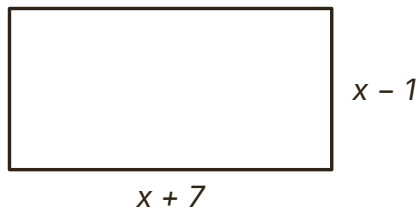
Find the coordinates of the turning point.



Not drawn accurately

Answer (.....,)

- D7** The diagram shows a rectangle. All lengths are in centimetres.
The area of the rectangle is 40 cm^2 .



Not drawn accurately

- (a) Show that $x^2 + 6x - 47 = 0$. [2 marks]

- (b) By completing the square, find the value of x . Give your answer in the form $a + b\sqrt{14}$. You must explain why there is only one answer. [4 marks]

Answer $x = \dots\dots\dots$

Total for D7: 6 marks

D8 $f(x) = x^2 - 6x + 17$

- (a) Write $f(x)$ in the form $(x + p)^2 + q$. [2 marks]

Answer

- (b) Hence explain why the equation $f(x) = 5$ has no real solutions. [2 marks]

Total for D8: 4 marks

D9 The curve $y = x^2 + 10x + 22$ is translated by the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. [4 marks]

Show that the new curve touches the x -axis, and find the point where it touches.

Answer (.....,)

D10 A ball is thrown upwards. Its height, h metres, after t seconds is given by

$$h = 30t - 5t^2$$

(a) Write h in the form $c - 5(t - d)^2$. [2 marks]

Answer

(b) Write down the greatest height of the ball, and the time at which it is reached. [2 marks]

Answer m after s

(c) For how many seconds is the ball more than 25 m above the ground? [2 marks]

Answer

Total for D10: 6 marks

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** Find the largest value that $\frac{1}{x^2 - 10x + 29}$ can take, and the value of x that gives it. [3 marks]

Answer

- E2** Find the values of k for which the equation $x^2 + 6x + k = 0$ has no real solutions. [3 marks]

Answer

- E3** The curve $y = x^2 - 2ax + 3a^2 + 4$, where a is a constant. [4 marks]

Find the coordinates of the turning point in terms of a , and show that the curve never meets the x -axis.

E4 The numbers x and y satisfy $x^2 + y^2 - 4x + 8y + 20 = 0$.

[4 marks]

Find the value of xy .

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write a quadratic in the form $(x + p)^2 + q$, including when the coefficient of x^2 is not 1.	☐	☐	☐
I can solve a quadratic exactly by completing the square.	☐	☐	☐
I can find the turning point and the minimum value from completed-square form.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

- E1**
- Hint 1** A fraction with 1 on top is largest when its bottom is smallest.
- Hint 2** $x^2 - 10x + 29 = (x - 5)^2 + 4$
- Hint 3** The bottom is never less than 4, and it equals 4 when $x = 5$. So the largest value is $\frac{1}{4}$.
-
- E2**
- Hint 1** Complete the square, then move the number to the other side.
- Hint 2** $(x + 3)^2 = 9 - k$
- Hint 3** A square is never negative, so there is no solution when $9 - k < 0$, which is when $k > 9$.
-
- E3**
- Hint 1** Treat a as a number. Halve the coefficient of x in the usual way.
- Hint 2** $y = (x - a)^2 - a^2 + 3a^2 + 4$
- Hint 3** $y = (x - a)^2 + 2a^2 + 4$, so the turning point is $(a, 2a^2 + 4)$. Since $2a^2 + 4$ is at least 4, y is always at least 4.
-
- E4**
- Hint 1** Complete the square twice: once for the x terms and once for the y terms.
- Hint 2** $(x - 2)^2 + (y + 4)^2 = 0$
- Hint 3** Two squares that add to 0 must both be 0. So $x = 2$ and $y = -4$.