



Recurring decimals, fractional powers, surds, bounds and counting

Grades 4–9 About 85 minutes No calculator
90 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 Write $\frac{3}{8}$ as a decimal.

[1 mark]

Answer

A2 Write $5^3 \times 5^4$ as a single power of 5.

[1 mark]

Answer

A3 Write 72 as a product of its prime factors. Give your answer in index form.

[2 marks]

Answer

A4 Round 7.0483 to 3 significant figures.

[1 mark]

Answer

A5 Write down the factors of 48 that are square numbers.

[1 mark]

Answer

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

Write $0.\dot{2}\dot{4}$ as a fraction in its simplest form.

Hint Call the decimal x . Two digits repeat, so multiply by 100 and subtract x .

B1 Write $0.\dot{6}\dot{3}$ as a fraction in its simplest form. [2 marks]

Answer

EXAMPLE 2

Work out the value of $27^{-\frac{2}{3}}$.

Hint The negative sign means one over. The 3 on the bottom means a cube root, and the 2 on top means square it.

B2 Work out the value of $125^{-\frac{2}{3}}$. [2 marks]

Answer

EXAMPLE 3

Simplify $\sqrt{112} - \sqrt{28}$.

Hint Find the largest square factor of each number. Both surds become a multiple of $\sqrt{7}$.

B3 Simplify $\sqrt{99} + \sqrt{44}$. [2 marks]

Answer

EXAMPLE 4

A rectangle is 8.4 cm long and 5.2 cm wide. Both lengths are correct to 1 decimal place.

Work out the upper bound of the perimeter of the rectangle.

Hint Each length could be up to 0.05 cm more than given. A perimeter is a sum, so use the two upper bounds.

B4 A rectangle is 7.3 cm long and 4.6 cm wide. Both lengths are correct to 1 decimal place. [2 marks]

Work out the lower bound of the perimeter of the rectangle.

Answer

C Practice

Grades 5–7

Each question asks for something different. Show your working.

C1 Here are three fractions.

[2 marks]

$$\frac{7}{40} \quad \frac{5}{12} \quad \frac{9}{75}$$

Which of them can be written as a terminating decimal? Give a reason for your answer.

Answer

C2 Write $0.\dot{3}1\dot{8}$ as a fraction in its simplest form.

[3 marks]

Answer

C3 Work out the value of $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$.

[2 marks]

Answer

C4 Find the value of x when $9^x = \frac{1}{27}$.

[2 marks]

Answer $x = \dots\dots$

C5 Give each answer in its simplest exact form.

(a) Expand and simplify $(3 + \sqrt{5})(2 - \sqrt{5})$.

[2 marks]

Answer

(b) Rationalise the denominator of $\frac{12}{\sqrt{6}}$.

[2 marks]

Answer

Total for C5: 4 marks

C6 The mass of a parcel is 64 kg, correct to the nearest kilogram.

[2 marks]

Write down the error interval for the mass, m kg.

Answer $\dots\dots \leq m < \dots\dots$

C7 A number y is **truncated** to 1 decimal place. The result is 7.2

[2 marks]

Write down the error interval for y .

Answer $\dots\dots \leq y < \dots\dots$

- C8** A car park ticket has a code made of two letters followed by three digits. The letters are from A to Z and the digits from 0 to 9. Letters and digits may be repeated. [2 marks]

How many different codes are possible?

Answer

- C9** Three-digit numbers are made from the digits 1, 2, 3, 4, 5 and 6. No digit is used more than once in a number. [2 marks]

How many of these numbers are even?

Answer

D Exam-style questions

Grades 6–9

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** Prove algebraically that the recurring decimal $0.4\dot{5}$ can be written as $\frac{41}{90}$. [3 marks]

- D2** Ali says, " $64^{-\frac{1}{2}} = -8$, because the minus sign makes the answer negative." [2 marks]

Ali is wrong. Explain why, and work out the value of $64^{-\frac{1}{2}}$.

Answer

D3 Solve $4^{x+1} = 8^{x-1}$.

[3 marks]

You must show your working.

Answer $x = \dots\dots$

D4 Show that $(\sqrt{45} + \sqrt{20})^2 = 125$.

[3 marks]

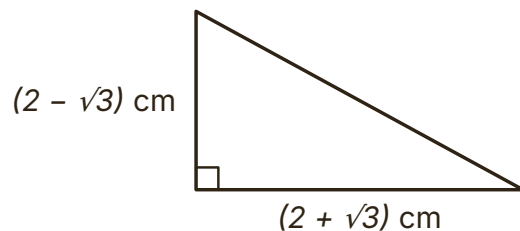
D5 $\frac{6}{2 - \sqrt{3}}$ can be written in the form $a + b\sqrt{3}$, where a and b are integers.

[4 marks]

Find the value of a and the value of b .

Answer $a = \dots\dots$ $b = \dots\dots$

D6 The diagram shows a right-angled triangle. The two shorter sides are $(2 + \sqrt{3})$ cm and $(2 - \sqrt{3})$ cm.



Not drawn accurately

(a) Show that the area of the triangle is $\frac{1}{2}$ cm².

[2 marks]

(b) Work out the length of the longest side. Give your answer as a surd.

[3 marks]

Answer

(c) Write the perimeter of the triangle in the form $a + \sqrt{b}$.

[1 mark]

Answer

Total for D6: 6 marks

D7 A bike lock opens with a code of four digits. Each digit is from 0 to 9.

(a) How many different codes are possible?

[1 mark]

Answer

(b) How many of the codes have four different digits?

[2 marks]

Answer

(c) Kai's code has four different digits. It starts with 7 and ends with an even digit.

[2 marks]

How many codes could Kai's code be?

Answer

Total for D7: 5 marks

D8 Simplify fully $(27x^6y^{-3})^{\frac{2}{3}}$.

[4 marks]

Answer

D9 A runner runs 70 m, correct to the nearest 10 m. She takes 13 seconds, correct to the nearest second.

(a) Work out the upper bound of her average speed. You must show your working. [3 marks]

Answer

(b) Show that her average speed could have been less than 5 m/s. [2 marks]

Total for D9: 5 marks

D10 Eight students stand for a club committee. One is chosen as captain, a different one as vice-captain and a third as secretary.

(a) In how many different ways can the three jobs be filled? [2 marks]

Answer

(b) Ella must be the captain. In how many ways can the other two jobs be filled? [1 mark]

Answer

Total for D10: 3 marks

D11 $\sqrt{2^x} \times 4 = 8^3$ [3 marks]

Find the value of x .

Answer $x = \dots\dots$

E Extension

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

E1 Show that $0.\dot{1}4285\dot{7} = \frac{1}{7}$.

[4 marks]

E2 Write $\frac{\sqrt{5}+1}{\sqrt{5}-1}$ in the form $\frac{a+\sqrt{b}}{c}$, where a , b and c are integers.

[3 marks]

Answer

E3 How many three-digit numbers have digits that add up to 6?

[3 marks]

Answer

E4 Solve $3^{2x} - 10 \times 3^x + 9 = 0$.

[4 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can tell which fractions terminate and change a recurring decimal into a fraction.	☐	☐	☐
I can work out negative and fractional powers, and solve equations with powers.	☐	☐	☐
I can simplify surds, expand brackets with surds and rationalise a denominator.	☐	☐	☐
I can write an error interval and choose the right bounds in a calculation.	☐	☐	☐
I can count outcomes with the product rule and check by listing.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

E1 **Hint 1** Six digits repeat. Multiply by a power of 10 that moves one whole block of them past the decimal point.

Hint 2 $1\,000\,000x - x = 142\,857$

Hint 3 $x = \frac{142\,857}{999\,999}$, and $7 \times 142\,857 = 999\,999$, so $x = \frac{1}{7}$.

E2 **Hint 1** To clear a surd from a bracket on the bottom, multiply top and bottom by the same bracket with the sign changed.

Hint 2
$$\frac{(\sqrt{5} + 1)(\sqrt{5} + 1)}{(\sqrt{5} - 1)(\sqrt{5} + 1)} = \frac{6 + 2\sqrt{5}}{4}$$

Hint 3 Divide every term by 2: $\frac{3 + \sqrt{5}}{2}$.

E3 **Hint 1** Work through the first digit, from 1 to 6, one at a time.

Hint 2 If the first digit is 1, the other two add to 5. There are 6 ways: 05, 14, 23, 32, 41, 50.

Hint 3 The counts for first digits 1 to 6 are 6, 5, 4, 3, 2, 1. Their total is 21.

E4 **Hint 1** 3^{2x} is $(3^x)^2$. Give 3^x a letter of its own.

Hint 2 With $y = 3^x$: $y^2 - 10y + 9 = 0$, so $(y - 1)(y - 9) = 0$.

Hint 3 $3^x = 1$ gives $x = 0$, and $3^x = 9$ gives $x = 2$.