



Relative frequency, expected outcomes, two-way tables and Venn diagrams

Grades 4–9 About 80 minutes No calculator
90 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 Write 0.35 as a fraction in its simplest form.

[1 mark]

Answer

A2 A bag holds 40 counters. $\frac{3}{8}$ of the counters are red. How many counters are not red?

[2 marks]

Answer

A3 Write $\frac{18}{48}$ in its simplest form.

[1 mark]

Answer

A4 A fair six-sided dice is rolled. Work out the probability that it lands on a number greater than 4.

[1 mark]

Answer

A5 Work out $1 - (0.45 + 0.28)$.

[1 mark]

Answer

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

A spinner is spun 120 times. It lands on red 42 times. Estimate the probability that the spinner lands on red.

Hint Relative frequency = number of reds $\div$ number of spins.

B1 A drawing pin is dropped 150 times. It lands point up 63 times. [2 marks]

Estimate the probability that the drawing pin lands point up.

Answer

EXAMPLE 2

The probability that a spinner lands on red is 0.35. The spinner is spun 240 times. Work out the expected number of times it lands on red.

Hint Expected number = probability $\times$ number of trials.

B2 A coin is biased. The probability that it lands on heads is 0.62. [2 marks]

The coin is thrown 350 times.

Work out the expected number of heads.

Answer

EXAMPLE 3

A spinner can land on red, blue, green or white. The probabilities are 0.15, 0.2, 0.3 and x . Work out the value of x .

Hint The four outcomes cannot happen together and one of them must happen, so the probabilities add up to 1.

B3 A spinner can land on red, blue, green or yellow. The table shows the probabilities. [3 marks]

Work out the probability that the spinner lands on blue.

Colour	Probability
Red	0.12
Blue	$2x$
Green	x
Yellow	0.28

Answer

EXAMPLE 4

There are 32 students in a class. 19 study French, 14 study Spanish and 6 study both. How many study neither?

Hint Put the 6 in the overlap first. Then French only is $19 - 6$ and Spanish only is $14 - 6$.

B4 In a group of 40 people, 23 have a cat, 15 have a dog and 4 have both a cat and a dog. [2 marks]

One of the people is chosen at random. Work out the probability that they have a cat but not a dog.

Answer

Each question asks for something different. Show your working.

C1 The two-way table shows how some students travel to school.

	Walk	Bus	Car	Total
Year 7	14	?	9	40
Year 8	?	12	?	35
Total	30	?	16	?

(a) How many Year 8 students travel by car?

[1 mark]

Answer

(b) One of the students is chosen at random. Work out the probability that this student travels by bus. [2 marks]

Answer

(c) One of the Year 8 students is chosen at random. Work out the probability that this student walks. [1 mark]

Answer

Total for C1: 4 marks

C2 Priya, Tom and Zoe each spin the same spinner. They record how many times it lands on red.

Name	Number of spins	Number of reds
Priya	40	15
Tom	100	31
Zoe	60	18

(a) Whose results give the most reliable estimate of the probability of red? Give a reason for your answer. [1 mark]

- (b) Use all of the results to estimate the probability that the spinner lands on red. [2 marks]

Answer

Total for C2: 3 marks

-
- C3** A card is taken at random from 30 cards numbered 1 to 30.

Event A : the number is a multiple of 4. Event B : the number is a square number. Event C : the number is odd.

- (a) Which two of the three events are mutually exclusive? [1 mark]

Answer

- (b) Work out the probability that the number is a multiple of 4 or an odd number. [2 marks]

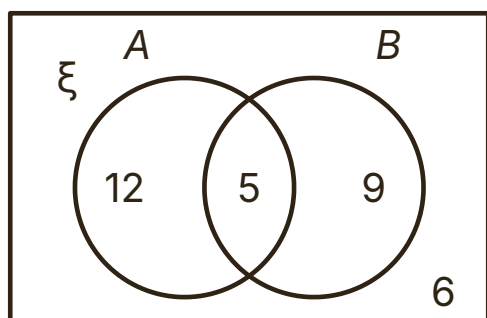
Answer

Total for C3: 3 marks

-
- C4** A fair six-sided dice is rolled a number of times. The expected number of sixes is 45. [2 marks]
How many times is the dice rolled?

Answer

C5 The Venn diagram shows the number of people in each region.



(a) One of the people is chosen at random. Work out $P(A \cup B)$.

[2 marks]

Answer

(b) Work out $P(A')$.

[2 marks]

Answer

Total for C5: 4 marks

C6 A bag contains only red, blue, green and white counters. A counter is taken at random. [3 marks]

The probability of red is 0.3. The probability of blue is 0.25. The probability of green is the same as the probability of white. There are 18 green counters in the bag.

Work out the total number of counters in the bag.

Answer

C7 The probability that a train is early is 0.08. The probability that it is on time is 0.71. Otherwise the train is late.

(a) Work out the probability that the train is late.

[1 mark]

Answer

- (b) The train runs on 300 days. Work out the number of days it is expected to be late. [2 marks]

Answer

Total for C7: 3 marks

- C8** A fair spinner is numbered 1, 2, 3 and 4. A second fair spinner is numbered 1, 2 and 3. [2 marks]
Both spinners are spun and the two numbers are added.
Work out the probability that the total is 5.

Answer

D Exam-style questions

Grades 6–9

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** 80 people at a gym are asked whether they prefer swimming or running. 45 of the people are women. 28 of the women prefer swimming. 22 of the men prefer running.

	Swimming	Running	Total
Women			45
Men		22	
Total			80

- (a) Use the information to complete the two-way table. [2 marks]
- (b) One of the people who prefer swimming is chosen at random. Work out the probability that this person is a man. [2 marks]

Answer

Total for D1: 4 marks

D2

Kai says, "The probability of an even number is $\frac{1}{2}$ and the probability of a multiple of 5 is $\frac{1}{5}$. So the probability of an even number or a multiple of 5 is $\frac{1}{2} + \frac{1}{5} = \frac{7}{10}$."

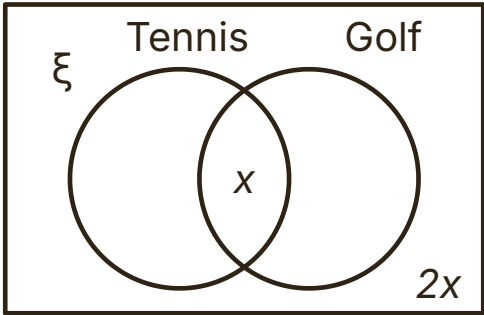
[3 marks]

A card is taken at random from 20 cards numbered 1 to 20. Explain why Kai is wrong, and work out the correct probability.

Answer

D3

There are 50 students in a club. 26 play tennis and 18 play golf. x students play both. $2x$ students play neither.



(a)

Work out the value of x .

[3 marks]

Answer

(b)

One of the students is chosen at random. Work out the probability that this student plays exactly one of the two sports.

[2 marks]

Answer

Total for D3: 5 marks

- D4** A bag contains 50 counters. Each counter is red or blue. Jen takes a counter at random, records its colour and puts it back. She does this 200 times. She records red 72 times.

(a) Estimate the number of red counters in the bag.

[2 marks]

Answer

(b) Explain why your answer to part (a) is only an estimate.

[1 mark]

Total for D4: 3 marks

- D5** A spinner can land on red, blue, green or yellow. The table shows the probabilities. The spinner is spun 400 times. [4 marks]

Work out the number of times the spinner is expected to land on green.

Colour	Red	Blue	Green	Yellow
Probability	$2x$	x	$x + 0.1$	0.22

Answer

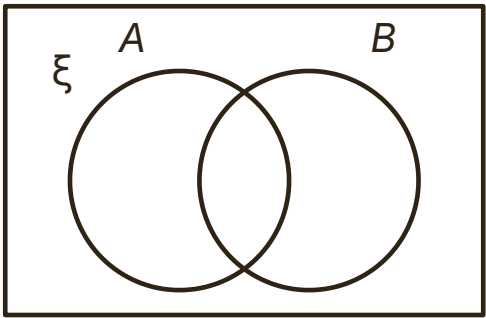
- D6** A school has 240 students in Year 10 and Year 11. 60% of the students are in Year 10. [6 marks]

$\frac{3}{8}$ of the Year 11 students walk to school. The number of Year 10 students who walk to school is 12 more than the number of Year 11 students who walk to school.

One of the students who walk to school is chosen at random. Work out the probability that this student is in Year 10.

Answer

D7 $\xi = \{\text{integers from 1 to 15}\}$. $A = \{\text{multiples of 3}\}$. $B = \{\text{factors of 30}\}$.



(a) List the members of $A \cap B$. [2 marks]

Answer

(b) A number is chosen at random from ξ . Work out $P(A \cup B)$. [2 marks]

Answer

Total for D7: 4 marks

D8 A spinner has five sections numbered 1 to 5. It is spun 200 times. The table shows the results.

Number	1	2	3	4	5
Frequency	38	41	44	37	40

(a) Estimate the probability that the spinner lands on an even number. [2 marks]

Answer

(b) The spinner is spun another 600 times. Estimate the number of times it lands on 3. [2 marks]

Answer

Total for D8: 4 marks

- D9** A box contains milk, dark and white chocolates. A chocolate is taken at random. The probability that it is milk is $\frac{5}{12}$. The probability that it is dark is $\frac{1}{4}$. [3 marks]
- There are 5 more milk chocolates than white chocolates. Work out the number of chocolates in the box.

Answer

- D10** It costs 50p to play a game at a stall. A player who wins is paid £2. The probability of winning is 0.15. [3 marks]
- 400 people play the game. Work out the profit the stall is expected to make.

Answer

E Extension

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** There are 50 people in a group. 30 like tea, 25 like coffee and 8 like neither. [3 marks]
- One of the people who like tea is chosen at random. Work out the probability that this person also likes coffee.

Answer

E2 A bag contains only red and green counters. The probability of taking a red counter is $\frac{3}{7}$. [3 marks]

6 more red counters are put in the bag. The probability of taking a red counter is now $\frac{1}{2}$.

How many counters were in the bag at the start?

Answer

E3 A fair spinner has n equal sections numbered 1 to n . It is spun 180 times. The expected number of times it lands on a prime number is 72. [3 marks]

Find all the possible values of n from 2 to 30.

Answer

E4 $\xi = \{\text{integers from 1 to 30}\}$. $A = \{\text{multiples of 4}\}$. $B = \{\text{multiples of 6}\}$. [3 marks]

A number is chosen at random from ξ . Work out $P(A \cup B)$.

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can estimate a probability from relative frequency, and say which estimate is more reliable.	☐	☐	☐
I can work out the expected number of times an outcome happens, and work back from it.	☐	☐	☐
I can use the fact that the probabilities of mutually exclusive, exhaustive outcomes add up to 1.	☐	☐	☐
I can find probabilities from two-way tables and Venn diagrams, including with set notation.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

-
- E1**
- Hint 1** First find how many people like tea or coffee or both.
- Hint 2** $50 - 8 = 42$ like at least one, so $30 + 25 - 42 = 13$ like both.
- Hint 3** Only the 30 tea drinkers can be chosen, so the probability is $\frac{13}{30}$.
-
- E2**
- Hint 1** Call the number of counters at the start n . Write the number of red counters in terms of n .
- Hint 2** At the start there are $\frac{3n}{7}$ red counters. Afterwards $\frac{3n}{7} + 6$ of the $n + 6$ counters are red.
- Hint 3** $2\left(\frac{3n}{7} + 6\right) = n + 6$ gives $\frac{n}{7} = 6$, so $n = 42$.
-
- E3**
- Hint 1** Work out the probability of a prime number first.
- Hint 2** $72 \div 180 = 0.4$, so the number of primes up to n must be $0.4n$.
- Hint 3** So n is a multiple of 5. Check $n = 5, 10, 15, 20, 25, 30$: the primes up to them number 3, 4, 6, 8, 9, 10. Only $n = 10, 15, 20$ work.
-
- E4**
- Hint 1** Count each set, then find the numbers in both.
- Hint 2** A has 7 members and B has 5. The numbers in both are the multiples of 12: 12 and 24.
- Hint 3** $A \cup B$ has $7 + 5 - 2 = 10$ members, so the probability is $\frac{10}{30} = \frac{1}{3}$.