



Factorising, rearranging formulae, algebraic fractions and completing the square

About 90 minutes No calculator 103 marks

Date _____

A Do Now

These bring back work from chapter 1 and from GCSE Higher that this chapter uses.

A1 Needed today [2 marks]
Factorise $x^2 - 11xy + 28y^2$.

Answer

A2 From chapter 1 [2 marks]
Expand and simplify $(x - 1)(x + 2)(x + 4)$.

Answer

A3 From chapter 1 [2 marks]
Work out $(5 + \sqrt{3})(5 - \sqrt{3})$.

Answer

A4 GCSE Higher [2 marks]
Make x the subject of $5(x - y) = 2y + 7$.

Answer

A5

GCSE Higher

[2 marks]

Solve $x^2 - 6x = 16$.

Answer

B Examples, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1Make x the subject of $y = \frac{3x - 1}{x + 2}$.**Hint** Multiply both sides by $x + 2$. Then collect every x term on one side and take x out as a common factor.**B1** Make a the subject of [3 marks]

$$k = \frac{5a + 2}{4 - a}$$

Answer

EXAMPLE 2Solve $\frac{2}{x - 1} + \frac{3}{x + 5} = 1$.**Hint** Multiply every term by $(x - 1)(x + 5)$. You get a quadratic: put everything on one side.**B2** Solve $\frac{3}{x - 2} + \frac{6}{x + 4} = 1$. [4 marks]

Answer

EXAMPLE 3

Write $5 + 12x - 3x^2$ in the form $a - b(x - c)^2$.

Hence write down the greatest value of $5 + 12x - 3x^2$.

Hint Take out -3 from the x terms: $5 - 3(x^2 - 4x)$. Complete the square inside the bracket.

B3 Write $1 + 12x - 2x^2$ in the form $a - b(x - c)^2$. [4 marks]

Hence write down the greatest value of $1 + 12x - 2x^2$.

Answer

EXAMPLE 4

Factorise fully $x^4 - 37x^2 + 36$.

Hint Treat x^2 as one letter: $x^4 - 37x^2 + 36$ is like $y^2 - 37y + 36$. Then look for the difference of two squares.

B4 Factorise fully $2x^4 - 10x^2 + 8$. [3 marks]

Answer

C Practice

Level 2

Each question asks for something different. Show your working.

C1 Factorise $4xy - 6x + 10y - 15$. [2 marks]

Answer

C2 Factorise $12x^2 - x - 20$. [2 marks]

Answer

C3 Factorise fully $3x^3y - 27xy^3$. [2 marks]

Answer

C4 Use factorising to work out $63.5^2 - 36.5^2$ without a calculator. [2 marks]

Answer

C5 Make x the subject of $y = 4 + \frac{3}{x-2}$. [3 marks]

Answer

C6 Simplify fully $\frac{5x^2 + 13x - 6}{25x^2 - 4}$. [3 marks]

Answer

C7 Write $\frac{5}{x+3} - \frac{2}{x-1}$ as a single fraction in its simplest form. [3 marks]

Answer

C8 Solve $\frac{x+1}{x-2} = \frac{x+4}{x-1}$. [3 marks]

Answer

C9 This question is about $x^2 - 10x + 22$.

(a) Write $x^2 - 10x + 22$ in the form $(x + a)^2 + b$.

[2 marks]

Answer

(b) Hence solve $x^2 - 10x + 22 = 0$. Give your answers in surd form.

[2 marks]

Answer

Total for C9: 4 marks

D Exam-style questions

AQA exam style

Answer every question in the space given. Show your working: most of the marks are for method.

D1 Sam simplifies $\frac{x^2 + 6x}{x^2 - 36}$. He writes " $\frac{x^2 + 6x}{x^2 - 36} = \frac{6x}{-36} = -\frac{x}{6}$ ". [3 marks]

Sam is wrong. Explain why, and simplify $\frac{x^2 + 6x}{x^2 - 36}$ fully.

Answer

D2 Ali drives 120 km at an average speed of x km/h. He drives back the same 120 km at an average speed 20 km/h faster, and the journey back takes 1 hour less.

(a) Show that $x^2 + 20x - 2400 = 0$.

[3 marks]

(b) Solve $x^2 + 20x - 2400 = 0$ to work out Ali's average speed on the way out. [2 marks]

Answer

(c) Work out the time Ali's journey back takes.

[1 mark]

Answer

Total for D2: 6 marks

D3 $y = 2x^2 - 12x + 23$

(a) Write $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$.

[3 marks]

Answer

(b) Hence write down the coordinates of the turning point of the graph of $y = 2x^2 - 12x + 23$.

[1 mark]

Answer

(c) Hence solve $2x^2 - 12x + 23 = 13$.

[2 marks]

Answer

Total for D3: 6 marks

D4 $a = \frac{b + 2c}{b - c}$

(a) Make b the subject of the formula.

[3 marks]

Answer

(b) Hence work out the value of b when $a = 4$ and $c = 3$.

[1 mark]

Answer

Total for D4: 4 marks

D5 This question is about $\frac{3}{x-4} - \frac{24}{x^2-16}$.

(a) Show that $\frac{3}{x-4} - \frac{24}{x^2-16} = \frac{3}{x+4}$. [3 marks]

(b) Hence solve $\frac{3}{x-4} - \frac{24}{x^2-16} = \frac{1}{2}$. [1 mark]

Answer

Total for D5: 4 marks

D6 n is an integer. [3 marks]

Show that $(2n+5)^2 - (2n+1)^2$ is a multiple of 8.

D7 This question is about $x^3 + 2x^2 - 9x - 18$.

(a) Factorise fully $x^3 + 2x^2 - 9x - 18$. [3 marks]

Answer

(b) Hence solve $x^3 + 2x^2 - 9x - 18 = 0$. [1 mark]

Answer

Total for D7: 4 marks

D8 Solve $2x^2 + 6x - 3 = 0$ by completing the square.

[4 marks]

Give your answers in the form $\frac{a \pm \sqrt{b}}{c}$, where a , b and c are integers.

Answer

D9 $16x^2 - 40x + k \equiv (px + q)^2 + 3$, where $p > 0$.

[3 marks]

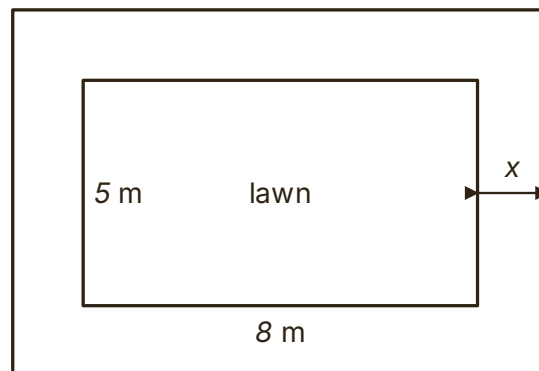
Work out the values of p , q and k .

Answer $p = \dots\dots$ $q = \dots\dots$ $k = \dots\dots$

D10 A rectangular lawn is 8 m long and 5 m wide. A path of width x m goes all the way round the lawn. The area of the path is 30 m^2 .

(a) Show that $2x^2 + 13x - 15 = 0$. [2 marks]

(b) Work out the width of the path. [2 marks]



Answer

Not drawn accurately

Total for D10: 4 marks

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

E1 x and y are positive whole numbers and $x^2 - y^2 = 15$. [3 marks]

Work out every pair of values of x and y .

Answer

E2 k is a constant. The expression $x^2 + 2kx + 9$ is positive for every value of x . [3 marks]

Work out the range of possible values of k .

Answer

E3 Simplify fully $\frac{16x^4 - 1}{8x^3 + 4x^2 + 2x + 1}$. [4 marks]

Answer

E4 $a > 1$ and $a + \frac{1}{a} = 3$. [4 marks]

Work out the value of $a^2 + \frac{1}{a^2}$. Hence work out the exact value of $a - \frac{1}{a}$.

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can factorise quadratics, quartics in x^2 , four terms by grouping and the difference of two squares.	☐	☐	☐
I can change the subject of a formula, including when the subject appears twice.	☐	☐	☐
I can simplify, add, subtract and solve equations with algebraic fractions.	☐	☐	☐
I can complete the square and use it to solve equations and find greatest and least values.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

E1	Hint 1	Factorise the left-hand side.
	Hint 2	$(x - y)(x + y) = 15$, and $x + y$ is the bigger factor. The factor pairs of 15 are 1×15 and 3×5 .
	Hint 3	$x - y = 1$, $x + y = 15$ gives $x = 8$, $y = 7$. $x - y = 3$, $x + y = 5$ gives $x = 4$, $y = 1$.
E2	Hint 1	Complete the square, keeping k as a letter.
	Hint 2	$x^2 + 2kx + 9 = (x + k)^2 + 9 - k^2$. The smallest value of $(x + k)^2$ is 0.
	Hint 3	The expression is always positive when $9 - k^2 > 0$, so $k^2 < 9$.
E3	Hint 1	Factorise the top as a difference of two squares, and the bottom by grouping the terms in pairs.
	Hint 2	$16x^4 - 1 = (4x^2 - 1)(4x^2 + 1)$ and $8x^3 + 4x^2 + 2x + 1 = 4x^2(2x + 1) + (2x + 1)$.
	Hint 3	$\frac{(2x - 1)(2x + 1)(4x^2 + 1)}{(2x + 1)(4x^2 + 1)}$. Cancel the common factors.
E4	Hint 1	Square both sides of $a + \frac{1}{a} = 3$.
	Hint 2	$\left(a + \frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2}$, so $a^2 + \frac{1}{a^2} = 7$.
	Hint 3	$\left(a - \frac{1}{a}\right)^2 = a^2 - 2 + \frac{1}{a^2} = 5$. When $a > 1$, $a - \frac{1}{a}$ is positive.