



Functions, composite and inverse functions, and graphs of linear, quadratic, exponential and piecewise functions

About 90 minutes No calculator 107 marks

Date _____

A Do Now

These bring back work from chapters 1 and 2 and from GCSE Higher that this chapter uses.

A1 Needed today [2 marks]

Work out the gradient of the line through $(-4, 5)$ and $(2, -4)$.

Answer

A2 From chapter 2 [2 marks]

Write $x^2 + 14x + 37$ in the form $(x + a)^2 + b$.

Answer

A3 From chapter 2 [2 marks]

Make x the subject of $y = \frac{x + 4}{3x - 2}$.

Answer

A4 GCSE Higher [2 marks]

Work out the value of $8^{-\frac{4}{3}}$.

Answer

A5

From chapter 1

[2 marks]

Expand and simplify $(2x - 3)^2 - (x - 3)^2$.

Answer

B Examples, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

$$f(x) = \frac{4x + 1}{x - 3} \text{ for } x > 3.$$

Work out $f^{-1}(x)$ and state the domain of f^{-1} .

Hint Write $y = \frac{4x + 1}{x - 3}$ and make x the subject.
The domain of f^{-1} is the range of f .

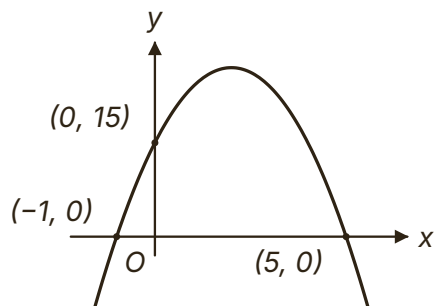
B1 $g(x) = \frac{2x + 5}{x + 1}$ for $x > -1$. [4 marks]

Work out $g^{-1}(x)$ and state the domain of g^{-1} .Answer $g^{-1}(x) = \dots\dots\dots$ domain $\dots\dots\dots$

EXAMPLE 2

The sketch shows the graph of $y = f(x)$, where $f(x)$ is a quadratic function.

Work out $f(x)$ in the form $ax^2 + bx + c$, and the coordinates of the maximum point.

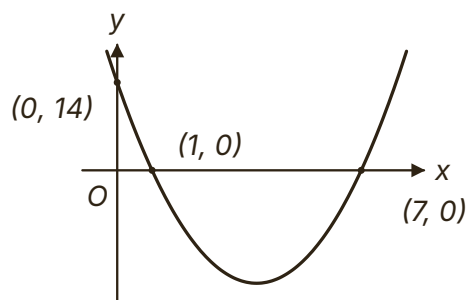


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Hint The roots give $f(x) = a(x + 1)(x - 5)$. Use the point on the y -axis to find a . The maximum point is half way between the roots.

B2 The sketch shows the graph of $y = g(x)$, where $g(x)$ is a quadratic function. [4 marks]

Work out $g(x)$ in the form $ax^2 + bx + c$, and the coordinates of the minimum point.



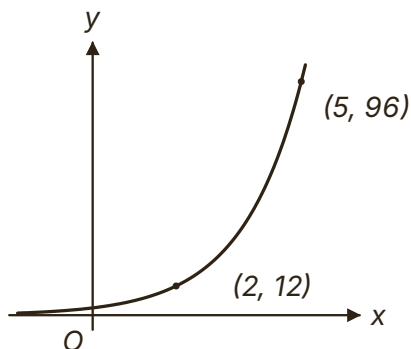
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Answer $g(x) = \dots\dots\dots$ ($\dots\dots, \dots\dots$)

EXAMPLE 3

The curve $y = ab^x$ passes through the points $(2, 12)$ and $(5, 96)$.

Work out the values of a and b . Then work out the value of y when $x = -1$.

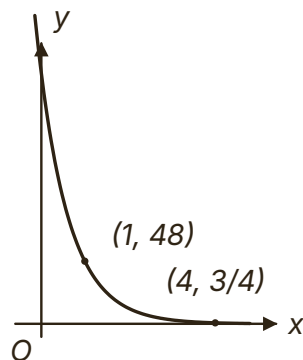


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Hint Write two equations, $ab^2 = 12$ and $ab^5 = 96$. Divide the second by the first so that a cancels.

B3 The curve $y = ab^x$ passes through the points $(1, 48)$ and $\left(4, \frac{3}{4}\right)$. [3 marks]

Work out the values of a and b . Then work out the value of y when $x = 3$.



Not drawn accurately

Answer $a = \dots\dots$ $b = \dots\dots$ $y = \dots\dots$

EXAMPLE 4

$$f(x) = \begin{cases} x^2 + a, & 0 \leq x < 2 \\ bx - 2, & 2 \leq x \leq 5 \end{cases}$$

The two parts of the graph of $y = f(x)$ join at $x = 2$, and $f(5) = 18$.

Work out the values of a and b , and the range of f .

Hint Use $f(5) = 18$ first: it is in the second part. Then make the two parts give the same value at $x = 2$.

B4 Here is a function f . [4 marks]

$$f(x) = \begin{cases} ax + 1, & -2 \leq x < 1 \\ x^2 + bx, & 1 \leq x \leq 4 \end{cases}$$

The two parts of the graph of $y = f(x)$ join at $x = 1$, and $f(4) = 8$.

Work out the values of a and b , and the range of f .

Answer $a = \dots$ $b = \dots$ range

c Practice

Level 2

Each question asks for something different. Show your working.

C1 $f(x) = x^2 - 5$. Solve $f(x + 1) = 2f(x)$. Give your answers in surd form. [3 marks]

Answer

C2 $f(x) = 5 - \sqrt{x + 3}$. Write down the largest possible domain of f , and the range of f for that domain. [2 marks]

Answer domain range

C3 $f(x) = \frac{x}{x - 1}$ for $x \neq 1$. Show that $ff(x) = x$. [3 marks]

C4 The line L has equation $3y = 12 - 4x$.

(a) Write down the gradient of L .

[1 mark]

Answer

(b) Work out the coordinates of the point on L where the y -coordinate is twice the x -coordinate. [2 marks]

Answer

Total for C4: 3 marks

C5 Work out the equation of the line through $(-2, 5)$ that is parallel to the line through $(1, -3)$ and $(5, 7)$. Give your answer in the form $px + qy + r = 0$, where p, q and r are integers.

[3 marks]

Answer

C6 The graph of $y = 2x^2 + px + q$ has line of symmetry $x = -3$ and passes through $(0, 11)$. Work out the values of p and q , and the least value of y .

[3 marks]

Answer $p = \dots\dots$ $q = \dots\dots$ least value $\dots\dots$

C7 $f(x) = \frac{1}{2}(x - 5)^2 + 2$ for $x \geq 5$. Work out an expression for $f^{-1}(x)$. Hence work out $f^{-1}(10)$.

[3 marks]

Answer $f^{-1}(x) = \dots\dots\dots$ $f^{-1}(10) = \dots\dots$

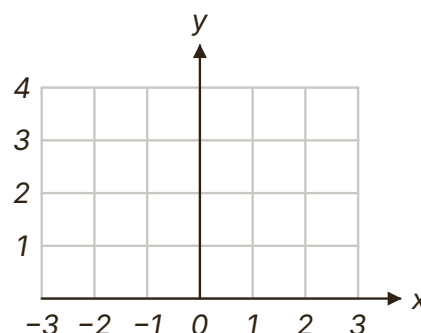
C8 The curves $y = 4^x$ and $y = 2^{x+3}$ meet at one point. Work out its coordinates. [2 marks]

Answer

C9
$$f(x) = \begin{cases} x + 3, & -3 \leq x < 0 \\ 3, & 0 \leq x < 2 \\ 9 - 3x, & 2 \leq x \leq 3 \end{cases}$$

(a) Draw the graph of $y = f(x)$ on the grid. [2 marks]

(b) Work out the area enclosed by the graph of $y = f(x)$ and the x -axis. [2 marks]



Answer

Total for C9: 4 marks

D Exam-style questions

AQA exam style

Answer every question in the space given. Show your working: most of the marks are for method.

D1 $f(x) = 3x - 4$. Kai says, " $f^{-1}(x) = \frac{1}{3x - 4}$ ". [3 marks]

Kai is wrong. Explain why, and work out $f^{-1}(11)$.

Answer

D2 $f(x) = 2x - 3$ for all values of x .

$g(x) = x^2 + 4$ for $x \geq 0$.

(a) Work out the value of $gf(3)$. [1 mark]

Answer

(b) Work out an expression for $g^{-1}(x)$.

[2 marks]

Answer

(c) Hence solve $fg^{-1}(x) = 7$.

[2 marks]

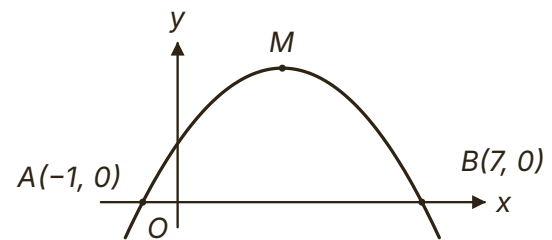
Answer

Total for D2: 5 marks

D3 The sketch shows the curve $y = -x^2 + px + q$.

It meets the x -axis at $A(-1, 0)$ and $B(7, 0)$. M is the maximum point of the curve.

(a) Work out the values of p and q . [2 marks]



Answer $p = \dots\dots$ $q = \dots\dots$

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(b) Hence work out the coordinates of M . [2 marks]

Answer

(c) Work out the equation of the line AM . Give your answer in the form $y = mx + c$.

[2 marks]

Answer

Total for D3: 6 marks

D4 The cost, $\mathcal{L}C$, of hiring a van for d days is a linear function of d . It costs $\mathcal{L}165$ to hire the van for 3 days and $\mathcal{L}325$ to hire it for 7 days.

(a) Work out a formula for C in terms of d .

[2 marks]

Answer

(b) Explain what the gradient of the graph of C against d represents. [1 mark]

(c) Ali has £500. Work out the greatest number of days he can hire the van for. [1 mark]

Answer

Total for D4: 4 marks

D5 The value of a car, £ V , t years after it is bought is modelled by $V = ab^t$, where a and b are constants.

After 1 year the car is worth £18 000. After 3 years it is worth £10 125.

(a) Work out the values of a and b . [3 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

(b) After how many complete years is the value of the car first less than £6000? You must show your working. [2 marks]

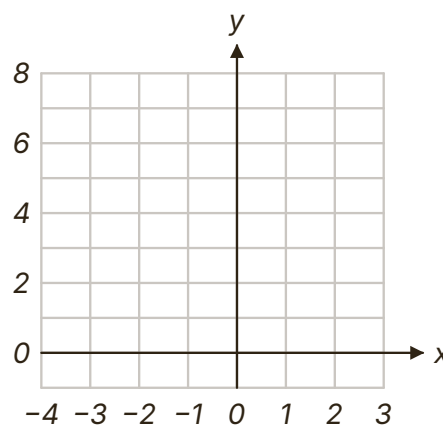
Answer

Total for D5: 5 marks

D6
$$f(x) = \begin{cases} x + 3, & -4 \leq x < -1 \\ x^2 + 1, & -1 \leq x < 1 \\ 3x - 1, & 1 \leq x \leq 3 \end{cases}$$

(a) Draw the graph of $y = f(x)$ on the grid. [2 marks]

(b) Work out the value of $ff(-2)$. [1 mark]



Answer

(c) Solve $f(x) = 0$. You must show that you have checked each part.

[2 marks]

Answer

(d) Work out the range of f .

[2 marks]

Answer

Total for D6: 7 marks

D7 The line L_1 has equation $2y = 3x + 8$. The line L_2 is parallel to L_1 and passes through $(4, 1)$.

(a) Work out the equation of L_2 . Give your answer in the form $y = mx + c$.

[2 marks]

Answer

(b) L_2 meets the line $y = x$ at P . Work out the coordinates of P .

[2 marks]

Answer

Total for D7: 4 marks

D8 $f(x) = \frac{12}{x+2}$ for $0 \leq x \leq 4$.

(a) Work out the range of f .

[2 marks]

Answer

(b) $g(x) = x^2 - 4x$ for all values of x .

[3 marks]

Hence work out the range of gf .

Answer

Total for D8: 5 marks

D9 $f(x) = x^2 + 10x + 27$ for $x \geq -5$.

(a) Write $f(x)$ in the form $(x + a)^2 + b$.

[2 marks]

Answer

(b) Hence write down the range of f .

[1 mark]

Answer

(c) Work out $f^{-1}(x)$ and state the domain of f^{-1} .

[3 marks]

Answer $f^{-1}(x) = \dots\dots\dots$ domain $\dots\dots\dots$

Total for D9: 6 marks

E Extension

Stretch

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

E1 $f(x) = x^2 - 2x$ for all values of x , and $g(x) = \frac{1}{x}$ for $x \neq 0$.

[3 marks]

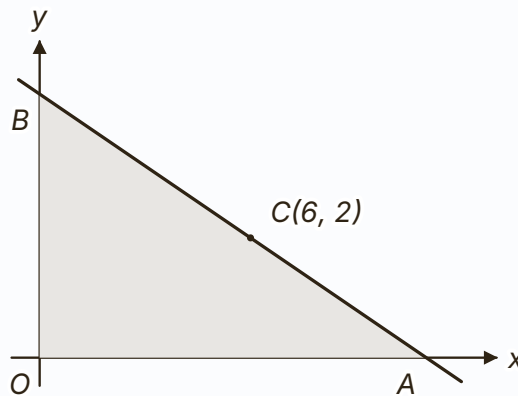
Work out the range of fg .

Answer

E2 A straight line with a negative gradient passes through $C(6, 2)$. It meets the positive x -axis at A and the positive y -axis at B . The area of triangle OAB is 24. [4 marks]

Work out the equation of the line. Give your answer in the form $px + qy = r$, where p, q and r are integers.

Answer



Not drawn accurately

E3 $f(x) = x^2 - 8x + 17$ for $x \geq 4$. [4 marks]
Solve $f(x) = f^{-1}(x)$. Give your answer in surd form.

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can use function notation, and find the domain and range of a function.	☐	☐	☐
I can form composite functions and solve equations with them.	☐	☐	☐
I can find inverse functions and state their domains.	☐	☐	☐
I can work with straight-line graphs and find the equation of a line.	☐	☐	☐
I can find the roots, turning point and equation of a quadratic graph.	☐	☐	☐
I can find the two constants of an exponential graph from two points on it, and use exponential models.	☐	☐	☐
I can draw and solve with functions defined in up to three parts.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

-
- E1** **Hint 1** Write $fg(x)$ in terms of x , then let $u = \frac{1}{x}$.
- Hint 2** $fg(x) = u^2 - 2u = (u - 1)^2 - 1$. The value of u can be any number except 0.
- Hint 3** $(u - 1)^2$ can be any number ≥ 0 (it is 0 when $u = 1$). Leaving out $u = 0$ does not lose a value, because $u = 2$ gives the same value as $u = 0$.
-
- E2** **Hint 1** Call the gradient $-m$, where $m > 0$. Write the coordinates of A and B in terms of m .
- Hint 2** $y - 2 = -m(x - 6)$, so $A = \left(6 + \frac{2}{m}, 0\right)$ and $B = (0, 2 + 6m)$.
- Hint 3** $\frac{1}{2} \left(6 + \frac{2}{m}\right) (2 + 6m) = 24$ gives $(6m + 2)^2 = 48m$, so $(6m - 2)^2 = 0$.
-
- E3** **Hint 1** f is increasing for $x \geq 4$, so the graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet on the line $y = x$.
- Hint 2** Solve $x^2 - 8x + 17 = x$, which is $x^2 - 9x + 17 = 0$.
- Hint 3** $x = \frac{9 \pm \sqrt{13}}{2}$. Only one of these is in the domain $x \geq 4$.
-