



Quadratics, simultaneous equations, the factor theorem, inequalities, indices, proof and sequences

About 90 minutes No calculator 116 marks

Date _____

A Do Now

The first is what this chapter needs. The rest bring back earlier work so nothing is forgotten.

A1 Needed today [2 marks]
Factorise $6x^2 - 5x - 21$.

Answer

A2 From chapter 2 [2 marks]
Write $x^2 + 8x + 23$ in the form $(x + a)^2 + b$.

Answer

A3 From chapter 3 [2 marks]
 $f(x) = 3x - 2$ and $g(x) = x^2 + 1$. Work out $gf(-1)$.

Answer

A4 From chapter 1 [2 marks]
Expand and simplify $(4 - \sqrt{7})^2$.

Answer

Solve the simultaneous equations $5x + 3y = 9$ and $3x - 4y = 17$.

Answer $x = \dots\dots$ $y = \dots\dots$

B Examples, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

Solve the simultaneous equations

$$x + 2y - z = 0$$

$$2x - y + 2z = 13$$

$$3x + y - z = 9$$

Hint Use two different pairs of equations to remove the same letter, z . That leaves two equations in x and y .

B1 Solve the simultaneous equations [5 marks]

$$2a + b - c = 9$$

$$a - 3b + 2c = -11$$

$$3a + 2b + c = 10$$

Do not use trial and improvement.

Answer $a = \dots\dots$ $b = \dots\dots$ $c = \dots\dots$

EXAMPLE 2

Solve the simultaneous equations

$$x + y = 3 \quad \text{and} \quad x^2 + 2y^2 = 18.$$

Hint Make y the subject of the linear equation and substitute it into the other one. Find both values of y at the end.

B2 Solve the simultaneous equations [5 marks]

$$y = 2x + 1$$

$$\text{and} \quad y^2 - 3x^2 = 6.$$

Answer $(\dots, \dots)$ and $(\dots, \dots)$

EXAMPLE 3

$f(x) = 2x^3 + ax^2 + bx + 6$. $(x - 2)$ and $(x + 3)$ are factors of $f(x)$.

Work out the values of a and b . Hence solve $f(x) = 0$.

Hint A factor $(x - 2)$ means $f(2) = 0$. That gives one equation in a and b ; $(x + 3)$ gives another.

B3 $f(x) = 3x^3 + ax^2 + bx - 8$. [5 marks]
 $(x + 2)$ and $(x - 1)$ are factors of $f(x)$.

Work out the values of a and b . Hence solve $f(x) = 0$.

Answer $a = \dots$ $b = \dots$ $x = \dots\dots$

EXAMPLE 4

Solve $2^{2x+1} - 9 \times 2^x + 4 = 0$.

Hint $2^{2x+1} = 2 \times (2^x)^2$. Let $y = 2^x$ and solve the quadratic in y .

B4 Solve $3^{2x+1} + 8 \times 3^x = 3$. [4 marks]

Answer $x = \dots\dots$

C Practice

Level 2

Each question asks for something different. Show your working.

C1 Solve $3x^2 + 6x - 2 = 0$. [3 marks]

Give your answers in the form $\frac{a \pm \sqrt{b}}{c}$, where a , b and c are integers.

Answer

- C2** At a cinema, 3 adult tickets and 5 child tickets cost £65.50. 2 adult tickets and 3 child tickets cost £41.50. [4 marks]

Work out the cost of 4 adult tickets and 1 child ticket.

Answer £

- C3** Show that $(2x - 3)$ is a factor of $2x^3 - 3x^2 - 8x + 12$. [4 marks]

Hence factorise $2x^3 - 3x^2 - 8x + 12$ fully.

Answer

- C4** Solve $x(x + 2) \geq 3(2x + 4)$. [3 marks]

Answer

- C5** $-2 \leq a \leq 3$ and $-5 \leq b \leq -1$.

- (a) Work out the greatest possible value of ab . [2 marks]

Answer

- (b) Work out the least possible value of $a^2 - b$. [2 marks]

Answer

Total for C5: 4 marks

C6 Write $\frac{(8x^6)^{\frac{2}{3}}}{2x^{-1}}$ in the form ax^n . [3 marks]

Hence solve $\frac{(8x^6)^{\frac{2}{3}}}{2x^{-1}} = 64$.

Answer $x = \dots\dots$

C7 Prove that the sum of the squares of any three consecutive integers is always 2 more than a multiple of 3. [3 marks]

C8 Work out an expression for the n th term of the quadratic sequence [3 marks]
2, 3, 10, 23, 42, ...

Answer

C9 The curve $y = ax^2 + bx + c$ passes through the points $(-1, 1)$, $(2, 4)$ and $(3, 17)$. [4 marks]
Work out the values of a , b and c .

Answer $a = \dots\dots$ $b = \dots\dots$ $c = \dots\dots$

D Exam-style questions

AQA exam style

Answer every question in the space given. Show your working: most of the marks are for method.

D1 Solve $\frac{6}{x+2} + \frac{1}{x-3} = 2$ [4 marks]

You must show your working.

Answer $x = \dots\dots$ or $x = \dots\dots$

D2 $f(x) = 2x^3 + 3x^2 - 11x - 6$. Kai works out $f(2) = 0$ and says, "So $x = -2$ is a solution of $f(x) = 0$."

Kai is wrong.

(a) Show that $x = -2$ is not a solution of $f(x) = 0$. [1 mark]

(b) Write down the factor of $f(x)$ that $f(2) = 0$ shows. [1 mark]

Answer

(c) Hence solve $f(x) = 0$. You must show your working. [4 marks]

Answer $x = \dots\dots\dots$

Total for D2: 6 marks

D3 Work out the integers x that satisfy both

[4 marks]

$$4x - 1 > x + 2 \quad \text{and} \quad 2x^2 - 3x - 9 \leq 0$$

You must show your working.

Answer

D4 Here is an expression.

$$\frac{9^x}{27^{x-1}}$$

(a) Write the expression in the form 3^k , where k is an expression in x .

[2 marks]

Answer

(b) Hence solve $\frac{9^x}{27^{x-1}} = \frac{1}{9}$

[2 marks]

Answer $x = \dots$

Total for D4: 4 marks

D5 Prove that the equation $x^2 + kx + k - 2 = 0$ has two different real roots for every value of k .

[4 marks]

D6 The n th term of a sequence is $\frac{6n-1}{2n+3}$

(a) A term of the sequence has the value 2.6. Work out which term it is.

[2 marks]

Answer

(b) Write down the limiting value of the sequence as $n \rightarrow \infty$. [1 mark]

Answer

(c) Show that every term of the sequence is less than 3. [2 marks]

Total for D6: 5 marks

D7 The n th term of a quadratic sequence is $an^2 + bn + c$.
The 1st term is 5, the 3rd term is 25 and the 5th term is 69.

(a) Work out an expression for the n th term. Do not use trial and improvement. [4 marks]

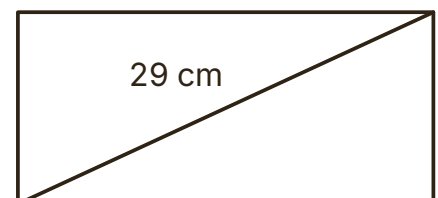
Answer

(b) Show that 265 is not a term of the sequence. [3 marks]

Total for D7: 7 marks

D8 A rectangle has a perimeter of 82 cm. Each diagonal of the rectangle is 29 cm long. [5 marks]

Work out the area of the rectangle. You must show your working.



Not drawn accurately

Answer cm²

D9 Circle the value of $64^{-\frac{2}{3}}$

[1 mark]

-16 $\frac{1}{16}$ $-\frac{128}{3}$ $\frac{1}{4}$

E Extension

Stretch

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

E1 Solve $2x^{\frac{1}{2}} - 7x^{\frac{1}{4}} + 3 = 0$.

[4 marks]

Answer $x = \dots\dots$ or $x = \dots\dots$

E2 The line $y = kx + 3$ is a tangent to the curve $y = x^2 - 3x + 12$.

[4 marks]

Work out the two possible values of k .

Answer $k = \dots\dots$ or $k = \dots\dots$

E3 Prove that $n^3 - n$ is a multiple of 6 for every positive integer n .

[3 marks]

E4 $f(x) = x^3 + ax^2 + bx + c$. $(x - 1)$ and $(x + 2)$ are factors of $f(x)$, and $f(2) = 20$. [5 marks]

Work out the values of a , b and c , and factorise $f(x)$ fully.

Answer $a = \dots$ $b = \dots$ $c = \dots$ $f(x) = \dots\dots\dots$

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can solve a quadratic equation, including one that starts as algebraic fractions.	☐	☐	☐
I can solve a linear and a quadratic equation together.	☐	☐	☐
I can use the factor theorem to find factors, unknown coefficients and roots.	☐	☐	☐
I can solve linear and quadratic inequalities and combine ranges.	☐	☐	☐
I can work with fractional and negative indices and solve equations in powers.	☐	☐	☐
I can write an algebraic proof.	☐	☐	☐
I can find the n th term and the limiting value of a sequence.	☐	☐	☐
I can solve three equations in three unknowns.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

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- E1** **Hint 1** $x^{\frac{1}{2}} = \left(x^{\frac{1}{4}}\right)^2$. Give $x^{\frac{1}{4}}$ a letter of its own.
- Hint 2** With $y = x^{\frac{1}{4}}$: $2y^2 - 7y + 3 = 0$, so $(2y - 1)(y - 3) = 0$.
- Hint 3** $x^{\frac{1}{4}} = \frac{1}{2}$ gives $x = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$, and $x^{\frac{1}{4}} = 3$ gives $x = 3^4 = 81$.
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- E2** **Hint 1** Put the two expressions for y equal. A tangent meets the curve at exactly one point.
- Hint 2** $x^2 - (3 + k)x + 9 = 0$ must have one repeated root.
- Hint 3** The discriminant is 0: $(3 + k)^2 - 36 = 0$, so $3 + k = 6$ or $3 + k = -6$, which gives $k = 3$ or $k = -9$.
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- E3** **Hint 1** Factorise $n^3 - n$ fully.
- Hint 2** $n^3 - n = (n - 1)n(n + 1)$, three consecutive integers.
- Hint 3** Of any two consecutive integers one is even, so the product is a multiple of 2. Of any three consecutive integers one is a multiple of 3. So the product is a multiple of $2 \times 3 = 6$.
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- E4** **Hint 1** Each fact gives one equation in a , b and c .
- Hint 2** $1 + a + b + c = 0$, $-8 + 4a - 2b + c = 0$, $8 + 4a + 2b + c = 20$.
- Hint 3** The last two give $4b = 4$, so $b = 1$. Then $a + c = -2$ and $4a + c = 10$, so $a = 4$ and $c = -6$. $f(x) = x^3 + 4x^2 + x - 6 = (x - 1)(x + 2)(x + 3)$.
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