

Algebraic fractions and equations with fractions

About 80 minutes No calculator 66 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-algebraic-fractions-and-equations-with-fractions/

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Work out $\frac{4}{9} \div \frac{10}{21}$. Give your answer in its simplest form.

Answer

A2 Needed today [2 marks]

Factorise $2x^2 - 7x - 15$.

Answer

- A3** Last lesson [2 marks]

Make y the subject of $x = \frac{2y-1}{y+3}$.

Answer

A4 From chapter 1 [2 marks]

Rationalise the denominator of $\frac{6}{\sqrt{5}-1}$. Simplify your answer.

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Simplify fully $\frac{3x+12}{x^2+x-12}$.

Hint Factorise the top and the bottom first. You can only cancel a whole bracket, never single terms.

- B1** Simplify fully $\frac{x^2-25}{3x^2+14x-5}$. [3 marks]

Answer

EXAMPLE 2

Simplify fully $\frac{8y-12x}{9x^2-6xy}$.

Hint After taking out common factors, one bracket is the other one backwards. Use $2y - 3x = -(3x - 2y)$.

- B2** Simplify fully $\frac{10b-15a}{6a^2-4ab}$. [2 marks]

Answer

EXAMPLE 3

Simplify $\frac{x^2-4}{x+3} \div \frac{x^2+x-6}{x^2+6x+9}$.

Hint Turn the second fraction upside down and multiply. Factorise everything before you cancel.

B3 Simplify $\frac{x^2+5x+4}{2x-6} \times \frac{x^2-9}{x^2+4x}$. [3 marks]

Answer

EXAMPLE 4

Write $\frac{x+2}{4} - \frac{2x-1}{6}$ as a single fraction in its simplest form.

Hint The lowest common denominator is 12. Put the second numerator in a bracket so the minus sign reaches every term.

B4 Write $\frac{3}{x+4} + \frac{2}{x-3}$ as a single fraction in its simplest form. [3 marks]

Answer

EXAMPLE 5

Write $\frac{3}{x+2} - \frac{12}{x^2-4}$ as a single fraction in its simplest form.

Hint $x^2 - 4 = (x + 2)(x - 2)$ already contains $x + 2$, so it is the common denominator. Do not multiply the two denominators together.

B5 Write $\frac{4}{x^2-2x-3} + \frac{1}{x+1}$ as a single fraction in its simplest form. [3 marks]

Answer

c Practice

Level 2

C1 Simplify fully $\frac{6x^2y^3}{9x^4y}$. [2 marks]

Answer

C2 Write $\frac{1}{x} + \frac{2}{x+1} - \frac{3}{x+2}$ as a single fraction in its simplest form. [3 marks]

Answer

C3 Solve $\frac{x+4}{3} - \frac{x-2}{5} = 2$.

[3 marks]

C4 Solve $\frac{3x-1}{4} + 2 = \frac{2x+5}{3}$.

[3 marks]

Answer

Answer

C5 Solve $\frac{5}{x-2} = \frac{3}{x+4}$. [2 marks]

C6 Write $\frac{4a^2b}{15c} \div \frac{6ab^2}{5c^2}$ as a single fraction in its simplest form. [2 marks]

Answer

Answer

D Exam-style questions

AQA exam style

D1 Mia says, " $\frac{2}{x+1} + \frac{3}{x+2} = \frac{5}{2x+3}$ ", because you add the tops and add the bottoms." [3 marks]

Show that Mia is wrong. Write $\frac{2}{x+1} + \frac{3}{x+2}$ as a single fraction correctly.

Answer

D2 Each part of this question uses the answer to the part before it.

(a) Factorise $6x^2 + x - 15$. [2 marks]

Answer

(b) Hence simplify fully $\frac{6x^2+x-15}{4x^2-9}$. [2 marks]

Answer

(c) Hence solve $\frac{6x^2+x-15}{4x^2-9} = 4$. [2 marks]

Answer

Total for D2: 6 marks

D3 This question is about the expression $\frac{1}{x-1} - \frac{1}{x+1}$.

(a) Show that $\frac{1}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$. [2 marks]

(b) Hence work out $\frac{1}{99} - \frac{1}{101}$. Give your answer as a fraction. [2 marks]

Answer

Total for D3: 4 marks

D4 Solve $\frac{6}{x+1} - \frac{1}{x-1} = 1$. You must show your working. [5 marks]

Answer

D5 A cyclist rides 30 km at x km/h, then a further 20 km at $(x + 5)$ km/h.

(a) Show that the total time, in hours, is $\frac{50x+150}{x(x+5)}$. [2 marks]

(b) The whole ride takes 3 hours. Show that $3x^2 - 35x - 150 = 0$. [2 marks]

(c) Work out the speed for the first 30 km. [2 marks]

Answer km/h

Total for D5: 6 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Work out the exact value of $\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{49 \times 51}$.

[5 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

E1 Hint 1 Each term is $\frac{1}{(2n-1)(2n+1)}$. Try writing it as a difference of two simpler fractions, the way $\frac{1}{x-1} - \frac{1}{x+1}$ worked in D3.

Hint 2 $\frac{1}{2n-1} - \frac{1}{2n+1} = \frac{2}{(2n-1)(2n+1)}$, so each term is $\frac{1}{2} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right)$.

Hint 3 Add them up: $\frac{1}{2} \left(\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \dots + \frac{1}{49} - \frac{1}{51} \right)$. Everything cancels except $\frac{1}{1}$ and $-\frac{1}{51}$, so the sum is $\frac{1}{2} \times \frac{50}{51} = \frac{25}{51}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can simplify an algebraic fraction by factorising and cancelling.



I can multiply and divide algebraic fractions.



I can add and subtract algebraic fractions with a common denominator.



I can solve equations that have fractions in them.

