



Circle theorems

About 75 minutes No calculator 57 marks

Date _____

A Do Now

The first is what today's lesson needs. The others bring back earlier topics.

- | | |
|---|--|
| <p>A1 Needed today [2 marks]</p> <p>An isosceles triangle has one angle of 38° between its two equal sides. Work out the size of each of the other two angles.</p> | <p>A2 Last lesson [2 marks]</p> <p>The angles of a quadrilateral are $(x + 20)^\circ$, $2x^\circ$, $(3x - 10)^\circ$ and $(2x + 30)^\circ$. Work out the value of x.</p> |
|---|--|

Answer

- | | |
|--|--|
| <p>A3 From chapter 5 [2 marks]</p> <p>A circle has equation $(x - 3)^2 + (y + 5)^2 = 49$. Write down the coordinates of its centre and its radius.</p> | <p>A4 From chapter 4 [2 marks]</p> <p>Solve the simultaneous equations $2x + y = 100$ and $x + 3y = 110$.</p> |
|--|--|

Answer

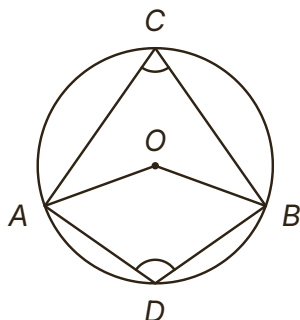
Answer

Answer

Your teacher works through each example with you. Then try the one beside it. O is always the centre.

EXAMPLE 1

O is the centre. Angle $AOB = 112^\circ$. Angle $ACB = x$ and angle $ADB = y$. Work out x and y . Give reasons.

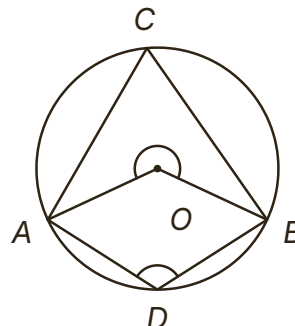


Not drawn accurately

Hint x and the 112° stand on the same arc AB . $ACBD$ is a cyclic quadrilateral.

B1 O is the centre. Angle [3 marks]

$ACB = 47^\circ$. Reflex angle $AOB = x$ and angle $ADB = y$. Work out x and y .

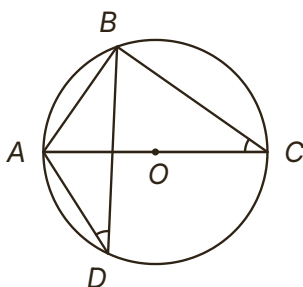


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Answer $x = \dots\dots^\circ$ $y = \dots\dots^\circ$

EXAMPLE 2

AC is a diameter. Angle $BAC = 31^\circ$. Angle $ACB = x$ and angle $ADB = y$. Work out x and y . Give reasons.

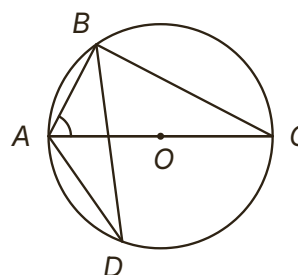


Not drawn accurately

Hint Angle ABC is in a semicircle. Then x and y stand on the same arc AB .

B2 AC is a diameter. Angle [3 marks]

$ADB = 38^\circ$ and angle $BAC = x$. Work out x .

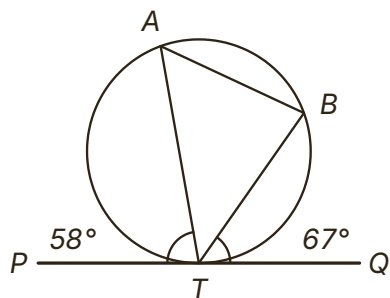


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Answer $x = \dots\dots^\circ$

EXAMPLE 3

PTQ is a tangent to the circle at T . Work out the three angles of triangle ATB . Give reasons.

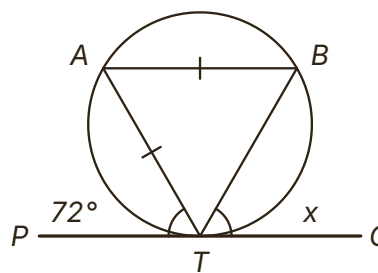


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Hint The angle between the tangent and the chord TA equals the angle in the other segment, at B .

B3 PTQ is a tangent to the circle [3 marks]

at T , and $TA = AB$. Work out angle x . Give reasons.



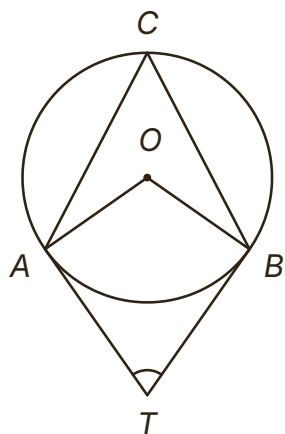
Not drawn accurately

Answer $x = \dots\dots^\circ$

c Practice

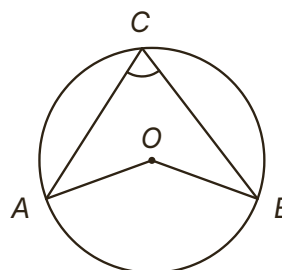
Level 2

- C1** TA and TB are tangents to the circle at A and B . Angle $ATB = 48^\circ$. Work out angle AOB and angle ACB . [3 marks]



Not drawn accurately

- C2** Angle $ACB = (x + 15)^\circ$ and reflex angle $AOB = (6x + 10)^\circ$. Work out the value of x . [3 marks]

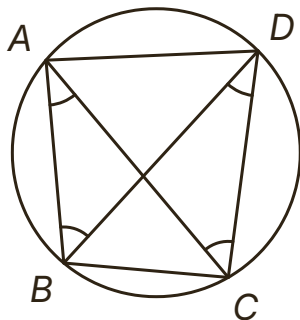


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Answer $x = \dots\dots$

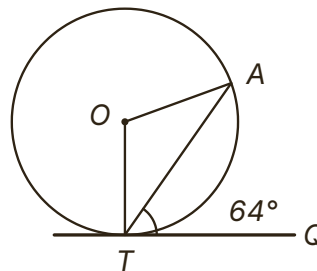
Answer $AOB = \dots\dots^\circ$ $ACB = \dots\dots^\circ$

- C3** A, B, C and D are points on the circle. Angle $BAC = 2x^\circ$, angle $BDC = (x + 24)^\circ$, angle $ABD = 3y^\circ$ and angle $ACD = (y + 40)^\circ$. Work out the values of x and y . [4 marks]



Not drawn accurately

- C4** TQ is a tangent to the circle at T . The angle between the tangent and the chord TA is 64° . Work out angle TOA . Give reasons. [3 marks]

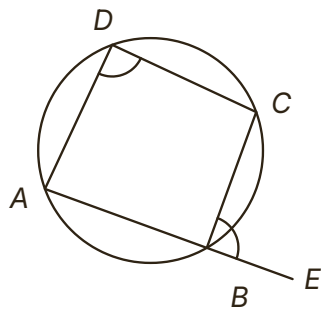


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Answer $^\circ$

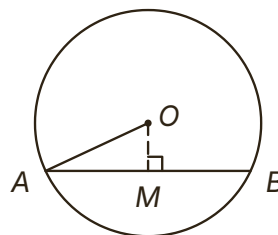
Answer $x = \dots\dots$ $y = \dots\dots$

- C5** $ABCD$ is a cyclic quadrilateral. ABE is a straight line. Angle $CBE = (x + 25)^\circ$ and angle $ADC = (3x - 15)^\circ$. Work out the size of angle ABC . [3 marks]



Not drawn accurately

- C6** The circle has radius 17 cm. The chord AB is 30 cm long. M is on AB and OM is perpendicular to AB . Work out the length OM . [2 marks]



Not drawn accurately

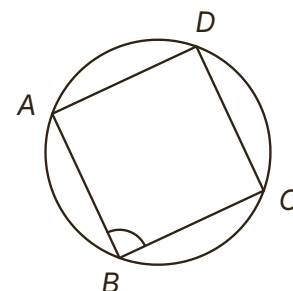
Answer cm

Answer $^\circ$

- D1** $ABCD$ is a cyclic quadrilateral with angle $ABC = 70^\circ$. Tom says, "Angle $ADC = 140^\circ$, because the angle at D is twice the angle at B ." [2 marks]

Tom is wrong. Work out the correct size of angle ADC and give the correct reason.

Answer $^\circ$



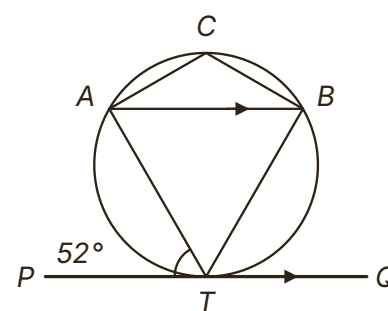
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- D2** PTQ is a tangent to the circle at T . AB is parallel to PQ . Angle $ATP = 52^\circ$. C is a point on the circle as shown.

- (a) Work out angle TBA . Give a reason. [2 marks]

Answer $^\circ$

- (b) Show that triangle TAB is isosceles. [2 marks]



Not drawn accurately

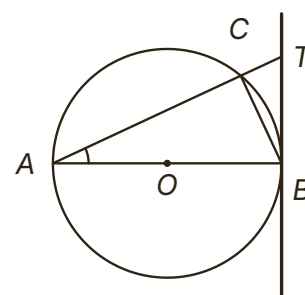
- (c) Hence work out angle ACB . Give a reason. [2 marks]

Answer $^\circ$

Total for D2: 6 marks

- D3** AB is a diameter. The tangent at B meets AC extended at T . Angle $CAB = 36^\circ$. [4 marks]
Work out angle CBT and angle BTC . Give reasons.

Answer $CBT = \dots\dots^\circ$ $BTC = \dots\dots^\circ$

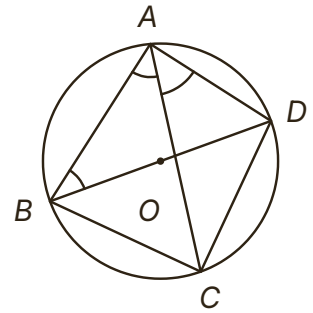


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D4 BD is a diameter. Angle $BAC = 3x^\circ$, angle $CAD = (2x - 10)^\circ$ and angle $ABD = (x + 30)^\circ$.

(a) Work out the value of x .

[3 marks]



Answer $x = \dots\dots$

(b) Work out angle ACB . Give reasons.

[2 marks]

Not drawn accurately

Answer $\dots\dots^\circ$

Total for D4: 5 marks

D5 O is the centre of a circle. A , B and C are points on the circle, with C on the major arc AB . Angle $AOB = 2y$. Which of these is angle ACB ? [1 mark]

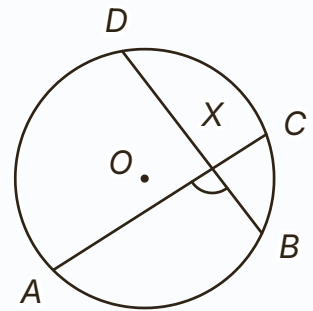
Circle your answer. $\frac{1}{2}y$ y $2y$ $180 - y$

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 O is the centre. The chords AC and BD cross at X . Angle $AOB = 100^\circ$ and angle $COD = 60^\circ$. Work out the size of angle AXB . Give reasons. [4 marks]



Answer $\dots\dots^\circ$

Not drawn accurately

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Find two angles of triangle XBC using angles at the circumference.
- Hint 2** Angle $ACB = 50^\circ$ (half of AOB) and angle $DBC = 30^\circ$ (half of COD).
- Hint 3** Angle AXB is the exterior angle of triangle XBC , so it is $50 + 30$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use the angle at the centre, semicircle and same segment theorems.



I can use cyclic quadrilaterals and the alternate segment theorem.



I can use tangents to a circle and give a reason for each step.

