



Combining transformations

About 80 minutes No calculator 70 marks

Date _____

A Do Now

The first question is what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}$. Work out $\mathbf{BA}$.

A2 Last lesson [2 marks]

Describe fully the single transformation given by the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

Answer

Answer

- A3** Last lesson [2 marks]

The matrix $\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$ maps the point P to the point $(4, 9)$. Work out the coordinates of P .

A4 From chapter 8 [2 marks]

Work out the equation of the tangent to the curve $y = x^2 - 4x + 1$ at the point where $x = 3$.

Answer (.....,)

Answer $y = \dots\dots$

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

The point $S(2, -1)$ is transformed by $\mathbf{P} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, followed by $\mathbf{Q} = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$. Work out the matrix for the combined transformation, and the image of S .

Hint The transformation done first goes on the right: the combined matrix is $\mathbf{QP}$, not $\mathbf{PQ}$.

- B1** The point $(-3, 2)$ is transformed by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$, followed by $\mathbf{B} = \begin{pmatrix} -1 & 3 \\ 2 & 1 \end{pmatrix}$. Work out the matrix for the combined transformation, and the image of the point. [3 marks]

Answer matrix = image = (.....,)

EXAMPLE 2

A rotation of 90° clockwise about the origin is followed by a reflection in the line $y = -x$. Work out the matrix for the combined transformation, and describe it as a single transformation.

Hint Write down the two matrices. The reflection is done second, so it goes on the left.

B2 A reflection in the x -axis is followed by a rotation of 270° anticlockwise about the origin. Work out the matrix for the combined transformation, and describe it as a single transformation. [4 marks]

Answer matrix =

EXAMPLE 3

A is a reflection in the line $y = x$ and **B** is a rotation of 90° anticlockwise about the origin. Work out **AB** and **BA**, and describe each.

Hint **BA** means **A** first, then **B**. Work out both products and compare.

B3 $\mathbf{G} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\mathbf{H} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$. Work out **GH** and **HG**. What do you notice? [3 marks]

Answer **GH** = **HG** =

EXAMPLE 4

$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Work out $\mathbf{R}^2$ and say what it means as a transformation. Hence write down $\mathbf{R}^4$.

Hint $\mathbf{R}^2 = \mathbf{RR}$. **R** is a quarter turn: what are two quarter turns?

B4 $\mathbf{S} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Describe the transformation **S** represents. Work out $\mathbf{S}^2$, and explain your answer using transformations. [3 marks]

Answer $\mathbf{S}^2 = \dots\dots$

EXAMPLE 5

A point Q is transformed by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$,
 then by $\mathbf{B} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, then by $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.
 Its final image is $(6, 3)$. Work out the
 coordinates of Q .

Hint The combined matrix is $\mathbf{CBA}$: last on the left.
 Work out $\mathbf{BA}$ first (or $\mathbf{CB}$ first, it gives the same),
 then solve.

B5 A point P is transformed by [4 marks]

$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$, then by
 $\mathbf{B} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, then by
 $\mathbf{C} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Its final image is
 $(4, 1)$. Work out the
 coordinates of P .

Answer $P = (\dots, \dots)$

c Practice

Level 2

C1 A rotation of 180° about the [3 marks]
 origin is followed by a reflection
 in the y -axis. Work out the
 matrix for the combined
 transformation, and describe it
 as a single transformation.

Answer matrix =

C2 Use matrices to show that an [3 marks]
 enlargement, scale factor -4 ,
 centre the origin, followed by an
 enlargement, scale factor $\frac{1}{2}$,
 centre the origin, is a single
 enlargement, centre the origin,
 with scale factor k . Write down
 the value of k .

Answer $k = \dots$

C3 A rotation of 90° anticlockwise [2 marks]
 about the origin is followed by a
 transformation $\mathbf{T}$. Every point
 ends where it started. Write
 down the matrix for $\mathbf{T}$ and
 describe $\mathbf{T}$.

Answer $\mathbf{T} = \dots$

C4 $\mathbf{P} = \begin{pmatrix} 2 & a \\ 1 & 3 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} b & 1 \\ 0 & -1 \end{pmatrix}$. [3 marks]
 The transformation $\mathbf{P}$ followed
 by the transformation $\mathbf{Q}$ has
 matrix $\begin{pmatrix} 7 & -3 \\ -1 & -3 \end{pmatrix}$. Work out the
 values of a and b .

Answer $a = \dots$ $b = \dots$

- C5** The point $(4, -7)$ is reflected in the line $y = -x$ and then rotated 90° anticlockwise about the origin. Use a single matrix to work out its final image. [3 marks]

Answer (.....,)

D Exam-style questions

AQA exam style

- D1** Transformation **A** is a reflection in the y -axis. Transformation **B** is a reflection in the x -axis. Which matrix represents **A** followed by **B**? Circle your answer. [1 mark]

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

- D2** Ravi says, "A rotation of 90° clockwise about the origin followed by a reflection in the y -axis gives the same as the reflection followed by the rotation. You multiply the same two matrices." [4 marks]

Use matrices to show that Ravi is wrong. Describe each of the two combined transformations.

- D3** **A**, **B** and **C** are transformations in the x - y plane. [4 marks]

A: a rotation of 180° about the origin

B: a reflection in the line $y = -x$

C: transformation **A** followed by transformation **B**

Use matrix multiplication to show that **C** is equivalent to a single reflection.

D4 $\mathbf{M} = \begin{pmatrix} 1 & k \\ 0 & 2 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$, where k is a constant.

- (a) Work out the matrix for $\mathbf{M}$ followed by $\mathbf{N}$. Give your answer in terms of k . [2 marks]

Answer

- (b) $\mathbf{M}$ followed by $\mathbf{N}$ maps the point $(3, 1)$ to the point $(14, 5)$. Work out the value of k . [2 marks]

Answer $k = \dots\dots$

- (c) Hence work out the point that $\mathbf{M}$ followed by $\mathbf{N}$ maps to $(12, 4)$. [3 marks]

Answer $(\dots\dots, \dots\dots)$

Total for D4: 7 marks

D5 Triangle T is shown on the grid. T is rotated 90° clockwise about the origin to give triangle U . U is then reflected in the x -axis to give triangle V .

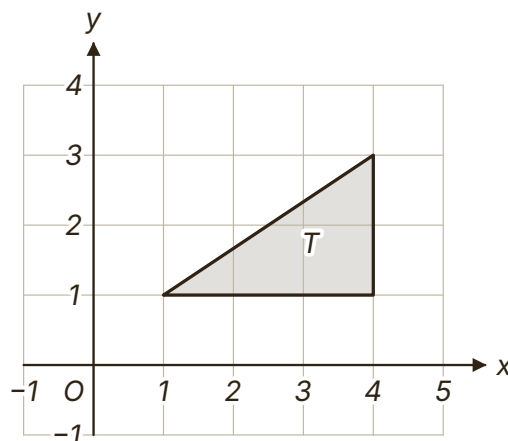
- (a) Work out the matrix that maps T to V . [3 marks]

Answer

- (b) Describe fully the single transformation that maps T to V . [1 mark]

Answer

- (c) Hence write down the coordinates of the vertices of V . [3 marks]



Answer

Total for D5: 7 marks

- D6** Matrix **P** represents a reflection in the x -axis. Matrix **Q** represents an enlargement, scale factor 4, centre the origin. The point A is transformed by **P** followed by **Q** to the point $(-8, 12)$. Work out the coordinates of A . [3 marks]

Answer (.....,)

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Transformation **A** is a reflection in the x -axis. Transformation **B** is a rotation of 90° anticlockwise about the origin. A point P is transformed by **A**, then **B**, then **A**, then **B**, and so on, taking turns: 2027 transformations in all, starting and ending with **A**. The final image is $(5, -2)$. Work out the coordinates of P . [5 marks]

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Work out the matrix for **A** followed by **B**. What does that pair do if it is done twice?
- Hint 2** $\mathbf{BA} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, a reflection in $y = x$, so two pairs do nothing. $2027 = 2 \times 1013 + 1$.
- Hint 3** There are 1013 pairs and one more **A**. 1013 is odd, so the pairs together are one **BA**. The whole chain is $\mathbf{A(BA)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Solve $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$: $y = 5$ and $x = 2$, so P is $(2, 5)$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can work out the single matrix for one transformation followed by another, multiplying in the right order. ☐ ☐ ☐
- I can describe a combined transformation as one single transformation. ☐ ☐ ☐
- I can show when the order of two transformations matters, and interpret a power such as $\mathbf{R}^2$. ☐ ☐ ☐
- I can work back from an image to the point it came from under a combined transformation. ☐ ☐ ☐