



Completing the square

About 75 minutes No calculator 62 marks

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Expand and simplify $(x - 7)^2 + 3$.

A2 Needed today [2 marks]

Expand and simplify $3(x + 2)^2 - 5$.

Answer

Answer

- A3** Last lesson [2 marks]

Write $\frac{3}{x-2} - \frac{2}{x+1}$ as a single fraction.

A4 From chapter 1 [2 marks]

How many four-digit even numbers can be made from the digits 1 to 9 if digits may be repeated?

Answer

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Write $x^2 + 10x + 7$ in the form $(x + a)^2 + b$.

Hint Halve the coefficient of x to get the number in the bracket. $(x + 5)^2$ makes $x^2 + 10x + 25$, so take away the extra 25.

- B1** Write $x^2 - 18x + 50$ in the form $(x + a)^2 + b$. [2 marks]

Answer

EXAMPLE 2

Write $x^2 + 3x - 1$ in the form $(x + a)^2 + b$.

Hint Half of 3 is $\frac{3}{2}$, and $(\frac{3}{2})^2 = \frac{9}{4}$. Keep the fractions; do not round.

- B2** Write $x^2 - 7x + 5$ in the form $(x + a)^2 + b$. [2 marks]

Answer

EXAMPLE 3

Write $2x^2 - 12x + 5$ in the form $a(x + b)^2 + c$.

Hint Take the 2 out of the x^2 and x terms only. Complete the square inside the bracket, then multiply back out.

B3 Write $3x^2 + 24x + 50$ in the form $a(x + b)^2 + c$. [3 marks]

Answer

EXAMPLE 4

Write $8 + 6x - x^2$ in the form $p - (x - q)^2$.

Hint Write it as $-(x^2 - 6x) + 8$. Complete the square inside, and watch the minus sign outside the bracket.

B4 Write $3 - 10x - x^2$ in the form $p - (x + q)^2$. [2 marks]

Answer

EXAMPLE 5

Work out the values of a , b and c such that $4x^2 + bx + 1 \equiv a(x - 2)^2 + c$.

Hint Expand the right-hand side. Then make the coefficients of x^2 , of x and the constants equal on both sides.

B5 Work out the values of a , b and c such that $5x^2 + bx - 3 \equiv a(x + 3)^2 + c$. [3 marks]

Answer

c Practice

Level 2

C1 Write $x^2 + 9x + 20$ in the form $(x + a)^2 + b$. [2 marks]

Answer

C2 Work out the values of c and d such that $x^2 - cx + 11 \equiv (x - 3)^2 + d$. [2 marks]

Answer

C3 Work out the values of a , b and c [3 marks]
such that
 $2x^2 + 5x - 1 \equiv a(x + b)^2 + c$.

Answer

C4 Work out the values of p , q and r [3 marks]
such that
 $9 - 8x - 2x^2 \equiv p - q(x + r)^2$.

Answer

C5 By completing the square, find [3 marks]
the minimum value of
 $x^2 + 4x + 9$ and the value of x
where it happens.

C6 Use completing the square to [3 marks]
make x the subject of
 $y = x^2 + 12x$, where $x > -6$.

Answer

Answer

D Exam-style questions

AQA exam style

D1 Ben writes " $2x^2 + 8x + 3 = 2(x + 2)^2 - 1$ ". [3 marks]

Explain Ben's mistake. Write $2x^2 + 8x + 3$ in the form $a(x + b)^2 + c$ correctly.

Answer

D2 This question is about $x^2 + 8x + 21$.

(a) Write $x^2 + 8x + 21$ in the form $(x + a)^2 + b$, where a and b are integers. [2 marks]

Answer

(b) Hence prove that $x^2 + 8x + 21$ is positive for all values of x . [2 marks]

(c) Hence write down the greatest value of $\frac{10}{x^2 + 8x + 21}$. [1 mark]

Answer

Total for D2: 5 marks

- D3** Write $23 - 20x - 4x^2$ in the form $a - (bx + c)^2$, where a , b and c are positive integers. [3 marks]

Answer

D4 $x^2 + px + q \equiv (x + 3)^2 - 7$

- (a) Work out the values of p and q . [2 marks]

Answer

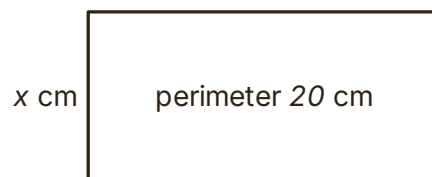
- (b) Hence solve $x^2 + px + q = 0$. Give your answers in surd form. [2 marks]

Answer

Total for D4: 4 marks

- D5** A rectangle has a perimeter of 20 cm. Its width is x cm.

- (a) Show that the area, A cm², of the rectangle is given by $A = 10x - x^2$. [2 marks]



Not drawn accurately

- (b) Write $10x - x^2$ in the form $p - (x - q)^2$. [2 marks]

Answer

- (c) Hence write down the greatest possible area of the rectangle, and the width that gives it. [2 marks]

Answer

Total for D5: 6 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Prove that $x^2 + y^2 - 4x + 6y + 14$ is positive for all values of x and y . Find its smallest value, and the values of x and y that give it. [5 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Collect the x terms together and the y terms together. Complete the square on each group separately.
- Hint 2** $x^2 - 4x = (x - 2)^2 - 4$ and $y^2 + 6y = (y + 3)^2 - 9$.
- Hint 3** So the expression is
 $(x - 2)^2 + (y + 3)^2 - 4 - 9 + 14 = (x - 2)^2 + (y + 3)^2 + 1$.
 Both squares are at least 0, so it is at least 1, which is positive. The smallest value is 1, when $x = 2$ and $y = -3$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write a quadratic in the form $(x + a)^2 + b$, including with fractions. ☐ ☐ ☐

I can complete the square when the coefficient of x^2 is not 1 or is negative. ☐ ☐ ☐

I can find unknown coefficients by comparing both sides of an identity. ☐ ☐ ☐

I can use a completed square to find a minimum or maximum, or to prove a quadratic is positive. ☐ ☐ ☐