



Composite functions

About 80 minutes No calculator 70 marks

Date _____

A Do Now

The first two questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks] **A2** Needed today [2 marks]
- $g(x) = \frac{12}{x+1}$. Solve $g(x) = 4$. Expand and simplify $(2x-3)^2 + 4$.

Answer $x = \dots\dots$

Answer $\dots\dots\dots$

- A3** Last lesson [2 marks] **A4** From chapter 2 [2 marks]
- $f(x) = 7 - 3x$ with domain $-2 \leq x \leq 4$. Write down the range of $f(x)$. Write $x^2 - 10x + 4$ in the form $(x-a)^2 - b$.

Answer $\dots\dots \leq f(x) \leq \dots\dots$

Answer $(x - \dots\dots)^2 - \dots\dots$

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$f(x) = 3x - 1$ and $g(x) = x^2 + 2$. Write $fg(x)$ and $gf(x)$ in their simplest forms.

Hint $fg(x)$ means $f(g(x))$: do g first, then put the result into f .

- B1** $f(x) = 2x + 5$ and $g(x) = x^2 - 3$. Write $fg(x)$ and $gf(x)$ in their simplest forms. [4 marks]

Answer $fg(x) = \dots\dots$ $gf(x) = \dots\dots$

EXAMPLE 2

$f(x) = 3x - 2$, $g(x) = \frac{1}{x}$ and $h(x) = x + 3$.
Write $fgh(x)$ and $ff(x)$ in their simplest forms.

Hint In $fgh(x)$ the function next to x works first: h , then g , then f . $ff(x)$ means $f(f(x))$.

B2 $f(x) = 2x - 1$, $g(x) = \frac{1}{x}$ and $h(x) = x^2$. Write $fgh(x)$ and $ff(x)$ in their simplest forms. [4 marks]

Answer $fgh(x) = \dots\dots$ $ff(x) = \dots\dots$

EXAMPLE 3

$h(x) = \frac{5}{x^2 + 1}$. Write $h(x)$ in the form $fg(x)$, stating $f(x)$ and $g(x)$.

Hint Ask what is done to x first. That step is g ; what is done to the result is f .

B3 $k(x) = 2(x - 3)^4$. Write $k(x)$ in the form $pqr(x)$, stating $p(x)$, $q(x)$ and $r(x)$. [3 marks]

Answer $p(x) = \dots\dots$ $q(x) = \dots\dots$ $r(x) = \dots\dots$

EXAMPLE 4

$f(x) = x^2 - 4$ and $g(x) = 2x + 1$. Solve $fg(x) = 21$.

Hint Write $fg(x)$ first. A square equal to 25 gives two solutions.

B4 $f(x) = x^2 + 3$ and $g(x) = 3x - 2$. Solve $fg(x) = 52$. [3 marks]

Answer $x = \dots\dots$ or $x = \dots\dots$

EXAMPLE 5

$f(x) = x - 2$ and $gf(x) = x^2 - 4x - 1$. Work out $g(x)$.

Hint Write $gf(x)$ in terms of $(x - 2)$, the output of f . Completing the square does it.

B5 $f(x) = x + 5$ and $gf(x) = x^2 + 10x + 30$. Work out $g(x)$. [3 marks]

Answer $g(x) = \dots\dots$

C Practice

Level 2

- C1** $f(x) = x^2 - 2x$ and $g(x) = 5 - x$. [3 marks]
Work out $fg(-1)$ and $gf(-1)$.

Answer $fg(-1) = \dots\dots$ $gf(-1) = \dots\dots$

- C2** $f(x) = x - 4$ and $g(x) = 2x$. [2 marks]
Write each of these in its simplest form: $f(x) \times g(x)$, then $fg(x)$.

Answer $f(x) \times g(x) = \dots\dots$ $fg(x) = \dots\dots$

- C3** $f(x) = \sqrt{x}$ for $x \geq 0$, and [2 marks]
 $g(x) = 10 - 2x$. Work out the domain of $fg(x)$.

- C4** $f(x) = x^2$ for all x , and [3 marks]
 $g(x) = 3 - x$ with domain $1 \leq x \leq 5$. Work out the range of $fg(x)$.

Answer $\dots\dots\dots$

Answer $\dots\dots \leq fg(x) \leq \dots\dots$

- C5** $f(x) = 2x + a$ and $g(x) = x^2 - 1$, [3 marks]
where a is a constant. Given that $fg(3) = 11$, find a . Then work out $gf(3)$.

Answer $a = \dots\dots$ $gf(3) = \dots\dots$

D Exam-style questions

AQA exam style

- D1** $f(x) = \sqrt{x}$ for $x \geq 0$. g is a function such that $gf(x) = 2x + 3\sqrt{x}$. Work out an expression for $g(x)$. [2 marks]

Answer $g(x) = \dots\dots$

- D2** $h(x) = 2x^2 - 7$. $k(x) = cx$, where c is a positive constant, and $hk(x) = 18x^2 - 7$. [3 marks]
Work out an expression for $kh(x)$.

Answer $kh(x) = \dots\dots$

D3 $f(x) = 2x^2 - 1$ for all x . $g(x) = x + 3$ for all x .

(a) Work out the value of $gf(2)$.

[2 marks]

Answer

(b) Show that $fg(x) = 2x^2 + 12x + 17$.

[2 marks]

(c) Hence solve $fg(x) = 13$. Give your answers in the form $p \pm \sqrt{q}$, where p and q are integers. [3 marks]

Answer $x = \dots \pm \sqrt{\dots}$

Total for D3: 7 marks

D4 $f(x) = x^2$ and $g(x) = x - 6$. Mia says, " $fg(x) = f(x) \times g(x)$, so $fg(x) = x^3 - 6x^2$."

Mia is wrong.

(a) Explain Mia's mistake, and write $fg(x)$ correctly.

[2 marks]

Answer $fg(x) = \dots$

(b) Solve $fg(x) = gf(x)$.

[3 marks]

Answer $x = \dots$

Total for D4: 5 marks

D5 $f(x) = 3x + p$ and $g(x) = x^2 + q$, where p and q are integers. $fg(x) = 3x^2 + 7$ and $gf(1) = 4$. Work out the values of p and q . You must show your working. [5 marks]

Answer $p = \dots$ $q = \dots$

D6 $f(x) = \frac{1}{x-2}$ for $x \neq 2$. $g(x) = 3x + 4$ for all x .

(a) Which value of x must be excluded from the domain of $fg(x)$? Circle your answer. [1 mark]

-2 $-\frac{2}{3}$ $\frac{2}{3}$ 2

(b) Solve $gf(x) = 10$. [2 marks]

(c) Show that $ff(x) = \frac{x-2}{5-2x}$. [3 marks]

Answer $x = \dots\dots$

Total for D6: 6 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $g(x) = ax + b$, where a and b are constants, and $gg(x) = 9x - 16$. Find every possible function $g(x)$. [4 marks]

Answer $g(x) = \dots\dots$ or $g(x) = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Work out $gg(x)$ in terms of a and b , then match it with $9x - 16$.
- Hint 2** $gg(x) = a(ax + b) + b = a^2x + ab + b$, so $a^2 = 9$ and $ab + b = -16$.
- Hint 3** $a = 3$ gives $4b = -16$, so $b = -4$. $a = -3$ gives $-2b = -16$, so $b = 8$. Both work.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write $fg(x)$, $gf(x)$, $ff(x)$ and $fgh(x)$ as expressions, and know the order matters. ☐ ☐ ☐

I can split a function into a composite of simpler functions. ☐ ☐ ☐

I can solve equations involving composite functions and find unknown constants. ☐ ☐ ☐

I can find a function from a composite, and the domain and range of a composite. ☐ ☐ ☐