



Lines, ratios, circles and tangents

About 90 minutes No calculator

122 marks

Date _____

A Do Now

The first is what this chapter needs. The rest bring back earlier work so nothing is forgotten.

A1 Needed today [2 marks]

Write $x^2 - 14x + 38$ in the form $(x - p)^2 - q$.

Answer

A2 From chapter 3 [2 marks]

A line passes through $(-3, 14)$ and $(2, -6)$. Work out its equation in the form $y = mx + c$.

Answer

A3 From chapter 4 [2 marks]

Solve $4x^2 - 4x - 15 = 0$.

Answer $x = \dots\dots$ or $x = \dots\dots$

A4 From chapter 1 [2 marks]

Simplify $\sqrt{98} + \sqrt{50}$.

Answer

Expand and simplify $(x - 3)^2 + (x + 1)^2$.

Answer

B Examples, then your turn

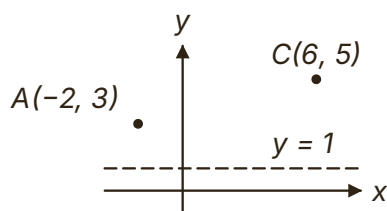
Level 2

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

A is $(-2, 3)$, B is $(k, 1)$ and C is $(6, 5)$.
Angle $ABC = 90^\circ$.

Work out the two possible values of k . Give your answers in surd form.

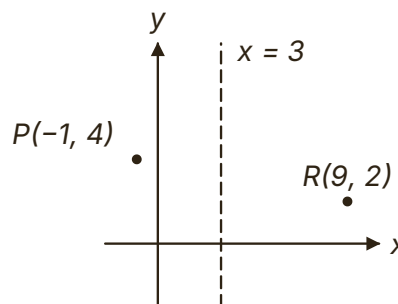


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Hint AB and BC are perpendicular, so their gradients multiply to -1 .

B1 P is $(-1, 4)$, Q is $(3, k)$ and R is $(9, 2)$. Angle $PQR = 90^\circ$. [4 marks]

Work out the two possible values of k .



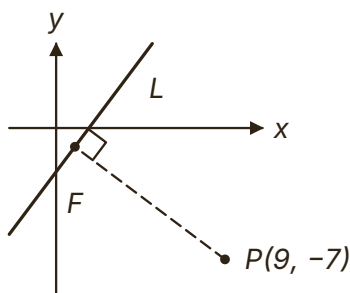
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Answer $k = \dots$ or $k = \dots$

EXAMPLE 2

P is the point $(9, -7)$. The line L has equation $4x - 3y = 7$.

Work out the coordinates of the foot of the perpendicular from P to L , and the shortest distance from P to L .

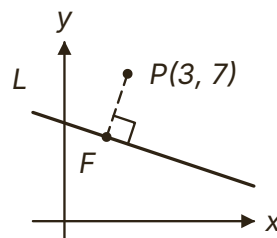


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Hint The line through P perpendicular to L meets L at the foot F . The shortest distance is PF .

B2 P is the point $(3, 7)$. The line L has equation $x + 3y = 14$. [5 marks]

Work out the coordinates of the foot of the perpendicular from P to L . Hence work out the shortest distance from P to L , as a surd.

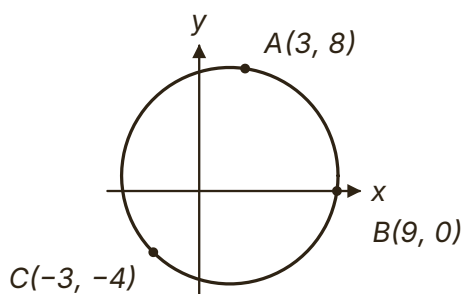


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Answer F $(\dots, \dots)$ distance $\dots$

EXAMPLE 3

A circle passes through the points $A(3, 8)$, $B(9, 0)$ and $C(-3, -4)$. Work out the equation of the circle.

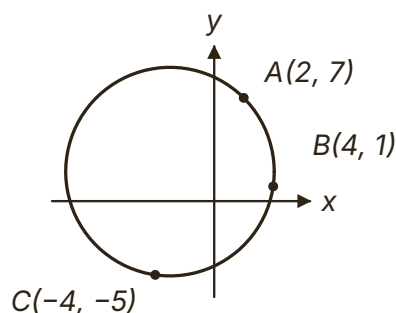


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Hint The centre is on the perpendicular bisector of every chord. Find two of them and where they meet.

B3 [5 marks]

A circle passes through the points $A(2, 7)$, $B(4, 1)$ and $C(-4, -5)$. Work out the equation of the circle.



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Answer

EXAMPLE 4

The line $y = 2x + c$ is a tangent to the circle $x^2 + y^2 - 2x + 4y - 15 = 0$. Work out the two possible values of c .

Hint Substitute $y = 2x + c$ into the circle. A tangent meets it once, so the quadratic in x has one repeated root.

B4 [5 marks]

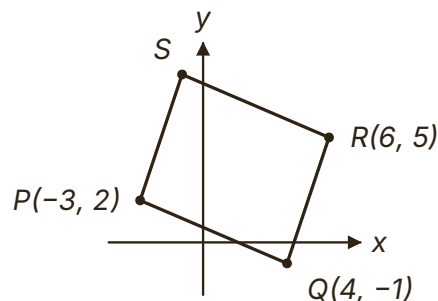
The line $y = 3x + c$ is a tangent to the circle $x^2 + y^2 + 4x - 6y + 3 = 0$. Work out the two possible values of c . You must show your working.

Answer $c = \dots$ or $c = \dots$

Each question asks for something different. Show your working.

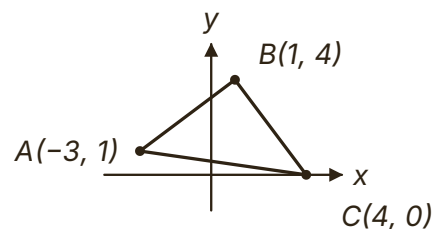
- C1** $PQRS$ is a parallelogram. P is $(-3, 2)$, Q is $(4, -1)$ and R is $(6, 5)$. [3 marks]
Work out the coordinates of S .

Answer (.....,



Not drawn accurately

- C2** A is $(-3, 1)$, B is $(1, 4)$ and C is $(4, 0)$. [4 marks]
Show that triangle ABC is right-angled and isosceles. Work out the area of the triangle.

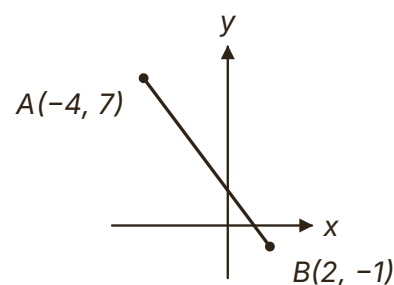


Answer area =

Not drawn accurately

- C3** A is $(-4, 7)$ and B is $(2, -1)$. Work out the equation of the perpendicular bisector of AB . Give your answer in the form $ax + by = c$, where a , b and c are integers. [3 marks]

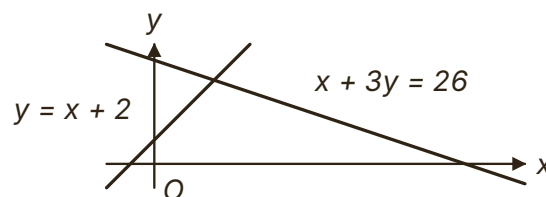
Answer



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- C4** The lines $y = x + 2$ and $x + 3y = 26$ and the x -axis enclose a triangle. [4 marks]

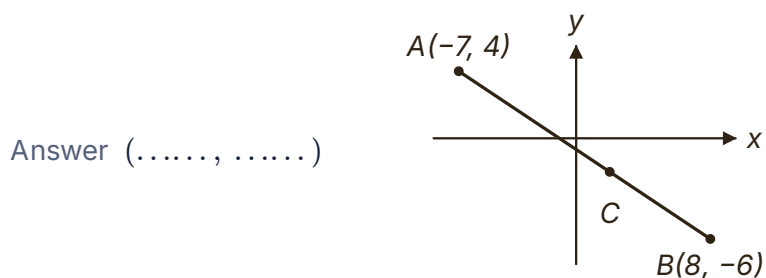
Work out the area of the triangle.



Answer square units Not drawn accurately

- C5** A is $(-7, 4)$ and B is $(8, -6)$. C is the point on AB with $AC : CB = 3 : 2$. [2 marks]

Work out the coordinates of C .



Answer (.....,)

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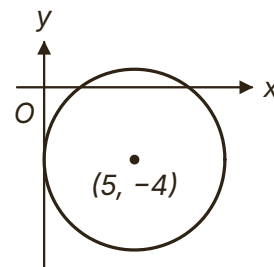
- C6** A circle has equation $x^2 + y^2 - 8x + 6y + 5 = 0$. [3 marks]

Work out the coordinates of its centre and its radius. Give the radius as a surd in its simplest form.

Answer centre (... , ...) radius

- C7** A circle has centre $(5, -4)$. The y -axis is a tangent to the circle. [3 marks]

Work out the coordinates of the points where the circle crosses the x -axis.



Answer (... , ...) and (... , ...)

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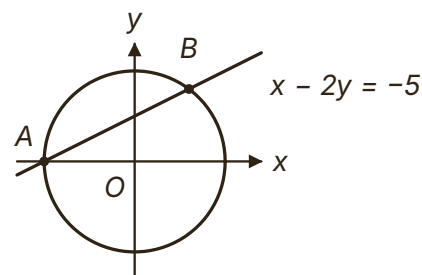
- C8** Show that the line $y = 2x + 8$ does not meet the circle $(x - 3)^2 + (y - 1)^2 = 16$.

[3 marks]

- C9** The line $x - 2y = -5$ meets the circle $x^2 + y^2 = 25$ at the points A and B .

[4 marks]

Work out the coordinates of A and B .



Answer $(\dots, \dots)$ and $(\dots, \dots)$

Not drawn accurately

D Exam-style questions

AQA exam style

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** Circle the centre of the circle $x^2 + y^2 - 6x + 10y = 2$

[1 mark]

$(-3, 5)$ $(3, -5)$ $(6, -10)$ $(-6, 10)$

- D2** Noah says, "The lines $4x + 5y = 7$ and $5x + 4y = 2$ are perpendicular, because the numbers in front of x and y have swapped over."

Noah is wrong.

- (a)** Show that the two lines are not perpendicular.

[2 marks]

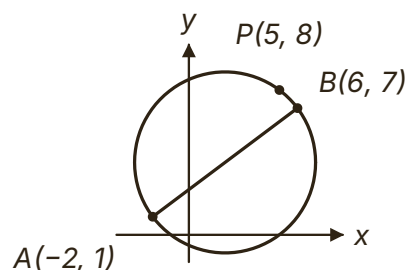
- (b) The line $5x + ky = 2$ is perpendicular to $4x + 5y = 7$. Work out the value of k . [2 marks]

Answer $k = \dots\dots$

Total for D2: 4 marks

D3 A is $(-2, 1)$ and B is $(6, 7)$. AB is a diameter of a circle.

- (a) Show that the equation of the circle is $x^2 + y^2 - 4x - 8y - 5 = 0$. [3 marks]



- (b) $P(5, 8)$ lies on the circle. Use gradients [2 marks] Not drawn accurately to show that angle $APB = 90^\circ$.

- (c) Work out the equation of the tangent to the circle at P . Give your answer in [3 marks] the form $ax + by = c$, where a, b and c are integers.

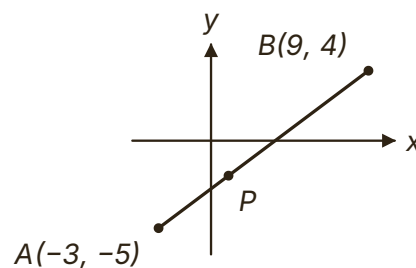
Answer $\dots\dots\dots$

Total for D3: 8 marks

D4 A is $(-3, -5)$ and B is $(9, 4)$. P is the point on AB with $AP : PB = 1 : 2$.

- (a) Work out the coordinates of P . [2 marks]

Answer $(\dots\dots, \dots\dots)$



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- (b) A circle has centre P and passes through A . Work out the equation of the circle. [2 marks]

Answer

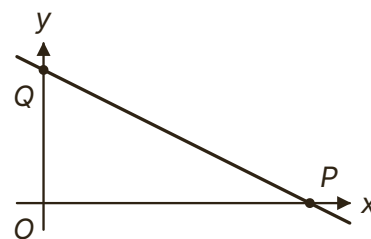
- (c) The circle crosses the y -axis at two points. Work out the exact distance between them. [3 marks]

Answer

Total for D4: 7 marks

D5 The line $x + 2y = 20$ meets the x -axis at P and the y -axis at Q .

- (a) Show that the perpendicular bisector of PQ has equation $2x - y = 15$. [3 marks]



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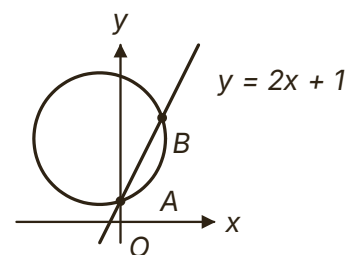
- (b) The perpendicular bisector meets the y -axis at R . Work out the area of triangle PQR . [2 marks]

Answer square units

Total for D5: 5 marks

D6 The line $y = 2x + 1$ meets the circle $x^2 + y^2 + 2x - 8y + 7 = 0$ at the points A and B .

- (a) Work out the exact length of AB . You must show your working. [5 marks]



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Answer

- (b) Hence work out the shortest distance from the centre of the circle to the line AB . [2 marks]

Answer

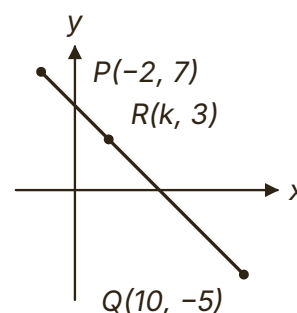
Total for D6: 7 marks

D7 P is $(-2, 7)$ and Q is $(10, -5)$. The point $R(k, 3)$ lies on the line PQ .

- (a) Work out the value of k . [2 marks]

Answer $k = \dots\dots$

- (b) Write down the ratio $PR : RQ$. [1 mark]



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Answer

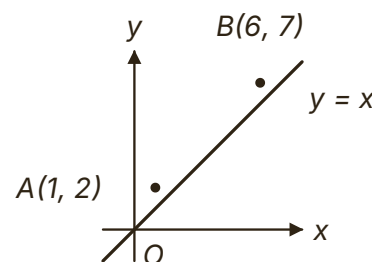
- (c) S lies on PQ extended beyond Q , with $PQ : QS = 3 : 1$. Work out the coordinates of S . [2 marks]

Answer $(\dots\dots, \dots\dots)$

Total for D7: 5 marks

D8 A is $(1, 2)$ and B is $(6, 7)$. The point C lies on the line $y = x$, and $AC = BC$.

- (a) Work out the coordinates of C . You must show your working. [4 marks]



Not drawn accurately

Answer $(\dots\dots, \dots\dots)$

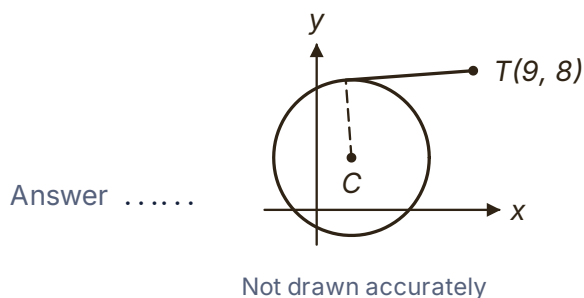
- (b) Work out the area of triangle ABC . [2 marks]

Answer square units

Total for D8: 6 marks

D9 A circle has equation $(x - 2)^2 + (y - 3)^2 = 20$. T is the point $(9, 8)$. [3 marks]

A tangent is drawn from T to the circle. Work out the length of the tangent. Give your answer as a surd in its simplest form.



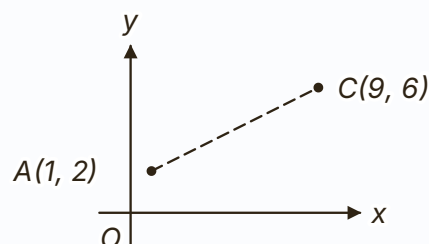
E Extension

Stretch

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

E1 $ABCD$ is a rhombus. A is $(1, 2)$ and C is $(9, 6)$. The vertex B lies on the x -axis. [5 marks]

Work out the coordinates of D and the area of the rhombus.



Answer D $(\dots, \dots)$ area

E2 The line $y = mx$ meets the circle $(x - 5)^2 + y^2 = 9$ at two different points. [4 marks]

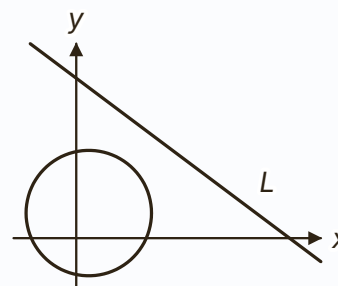
Work out the range of possible values of m .

Answer

E3 A circle has equation $x^2 + y^2 - 2x - 4y - 20 = 0$. The line L has equation $3x + 4y = 51$. [4 marks]

Work out the shortest distance from the circle to the line L .

Answer



Not drawn accurately

E4 A is the point $(0, 0)$ and B is the point $(6, 0)$. The point $P(x, y)$ moves so that $PA = 2PB$. [5 marks]

Show that P lies on a circle. Work out its centre and its radius.

Answer centre $(\dots, \dots)$ radius

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can use gradients, lengths and midpoints to prove facts about shapes, and find an unknown coordinate.	☐	☐	☐
I can find equations of parallel and perpendicular lines, where lines meet, and the shortest distance to a line.	☐	☐	☐
I can find a point that divides a line in a given ratio, inside or beyond the line.	☐	☐	☐
I can find the equation of a circle, and its centre and radius by completing the square.	☐	☐	☐
I can use tangents, chords and the angle in a semicircle, and find where a line meets a circle.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

E1	Hint 1	The diagonals of a rhombus bisect each other at right angles. So B and D are on the perpendicular bisector of AC .
	Hint 2	The perpendicular bisector of AC is $y - 4 = -2(x - 5)$. It meets the x -axis at $B(7, 0)$.
	Hint 3	The midpoint $(5, 4)$ is also the midpoint of BD , so D is $(3, 8)$. Area $= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times \sqrt{80} \times \sqrt{80} = 40$.
E2	Hint 1	Substitute $y = mx$ into the circle and collect the terms in x .
	Hint 2	$(1 + m^2)x^2 - 10x + 16 = 0$. Two different points means two different real roots: $b^2 - 4ac > 0$.
	Hint 3	$100 - 64(1 + m^2) > 0$ gives $m^2 < \frac{36}{64}$, so $-\frac{3}{4} < m < \frac{3}{4}$.
E3	Hint 1	Complete the square to find the centre C and the radius. The nearest point of the circle is on the line from C perpendicular to L .
	Hint 2	The circle is $(x - 1)^2 + (y - 2)^2 = 25$. The line through $C(1, 2)$ perpendicular to L is $4x - 3y = -2$.
	Hint 3	That line meets L at $(5.8, 8.4)$. The distance from C is $\sqrt{4.8^2 + 6.4^2} = 8$, so the shortest distance from the circle is $8 - 5 = 3$.
E4	Hint 1	Write PA^2 and PB^2 in terms of x and y . $PA = 2PB$ means $PA^2 = 4PB^2$.
	Hint 2	$x^2 + y^2 = 4((x - 6)^2 + y^2)$, which simplifies to $x^2 + y^2 - 16x + 48 = 0$.
	Hint 3	Complete the square: $(x - 8)^2 + y^2 = 16$. That is a circle with centre $(8, 0)$ and radius 4.