

Dividing a line in a given ratio

About 80 minutes No calculator 71 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-dividing-a-line-in-a-given-ratio/

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

M is the midpoint of AB . A is $(-2, 7)$ and M is $(3, 1)$. Work out the coordinates of B .

A2 Needed today [2 marks]

Work out the number that is $\frac{3}{8}$ of the way from -6 to 14 on a number line.

Answer (.....,)

Answer

- A3** Last lesson [2 marks]

Work out the coordinates of the point where the lines $2x + y = 7$ and $x - 3y = 14$ meet.

A4 From chapter 4 [2 marks]

Solve $2x^2 + x - 15 = 0$.

Answer $x = \dots\dots$ or $x = \dots\dots$

Answer (.....,)

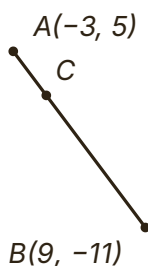
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A is $(-3, 5)$ and B is $(9, -11)$. C is the point on AB with $AC : CB = 1 : 3$. Work out the coordinates of C .



Not drawn accurately

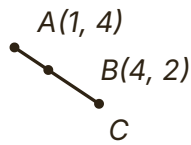
Hint The ratio has $1 + 3 = 4$ parts, so C is $\frac{1}{4}$ of the way from A to B .

- B1** A is $(-5, -2)$ and B is $(10, 8)$. [3 marks]
- C is the point on AB with $AC : CB = 2 : 3$. Work out the coordinates of C .

Answer (.....,)

EXAMPLE 2

ABC is a straight line. A is $(1, 4)$ and B is $(4, 2)$. $AB : BC = 2 : 3$. Work out the coordinates of C .



Not drawn accurately

Hint C is beyond B . BC is $\frac{3}{2}$ of AB , and goes the same way.

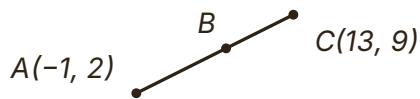
B2 ABC is a straight line. A is $(-2, -5)$ and B is $(1, 1)$. [3 marks]

$AB : BC = 3 : 2$. Work out the coordinates of C .

Answer (.....,)

EXAMPLE 3

ABC is a straight line. A is $(-1, 2)$ and C is $(13, 9)$. $AC : BC = 7 : 3$. Work out the coordinates of B .



Not drawn accurately

Hint AC is the whole line, 7 parts; BC is 3 of them. So AB is 4 parts and B is $\frac{4}{7}$ of the way from A .

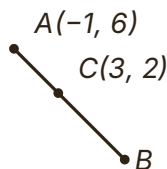
B3 ABC is a straight line. A is $(-4, -3)$ and C is $(14, 15)$. [3 marks]

$AC : BC = 9 : 4$. Work out the coordinates of B .

Answer (.....,)

EXAMPLE 4

C is the point on AB with $AC : CB = 2 : 3$. A is $(-1, 6)$ and C is $(3, 2)$. Work out the coordinates of B .



Not drawn accurately

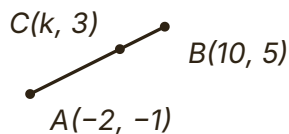
Hint A to C is 2 parts. One part is half of it. C to B is 3 parts.

B4 C is the point on AB with $AC : CB = 3 : 1$. A is $(5, -7)$ and C is $(-4, 5)$. Work out the coordinates of B . [3 marks]

Answer (.....,)

EXAMPLE 5

A is $(-2, -1)$ and B is $(10, 5)$. The point $C(k, 3)$ lies on AB . Work out k and the ratio $AC : CB$.



Not drawn accurately

Hint Use the y -coordinates: from -1 to 5 is 6 ; to reach 3 is 4 of those 6 .

B5 A is $(-3, 7)$ and B is $(9, -2)$. [4 marks]

The point $C(k, 4)$ lies on AB .
Work out the value of k and
the ratio $AC : CB$.

Answer $k = \dots\dots$ $AC : CB = \dots\dots$

c Practice

Level 2

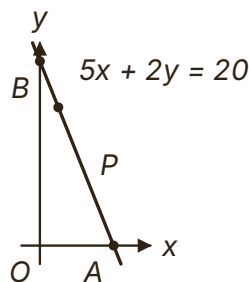
C1 DEF is a straight line. E is $(5, -1)$ and F is $(13, 11)$. [3 marks]
 $DE : EF = 3 : 4$. Work out the coordinates of D .

Answer $(\dots\dots, \dots\dots)$

C2 ABC is a straight line. AB is 50% longer than BC . A is $(-6, 9)$ and B is $(3, -3)$. (a) Work out the ratio $AB : BC$ in its simplest form. (b) Work out the coordinates of C . [4 marks]

Answer (a) $\dots\dots$ (b) $(\dots\dots, \dots\dots)$

- C3** The line $5x + 2y = 20$ meets the x -axis at A and the y -axis at B . P is the point on AB with $AP : PB = 3 : 1$. Work out the coordinates of P . [3 marks]



Not drawn accurately

- C4** PQR is a straight line. P is $(-2, a)$, Q is $(b, 4)$ and R is $(10, 13)$. $PQ : QR = 1 : 3$. Work out the values of a and b . [4 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

Answer $(\dots\dots, \dots\dots)$

- C5** A is $(-4, 1)$, B is $(2, 4)$ and C is $(10, 8)$. (a) Show that A , B and C lie on a straight line. (b) Work out the ratio $AB : BC$ in its simplest form. [3 marks]

Answer (b) $\dots\dots$

D Exam-style questions

AQA exam style

- D1** A is $(0, 0)$ and B is $(20, 15)$. C is the point on AB with $AC : CB = 1 : 4$. Circle the coordinates of C . $(4, 3)$ $(5, 3.75)$ $(10, 7.5)$ $(16, 12)$ [1 mark]

Answer

- D2** A is $(2, -3)$ and B is $(14, 6)$. C is on AB with $AC : CB = 2 : 1$. Jamal says C is $(6, 0)$, because $\frac{1}{3}$ of 12 is 4 and $\frac{1}{3}$ of 9 is 3. [3 marks]

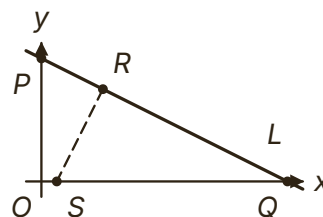
Explain Jamal's mistake. Work out the correct coordinates of C .

Answer $(\dots\dots, \dots\dots)$

- D3** ABC is a straight line with $AB : BC = 3 : 5$. B is $(4, 6)$ and C is $(14, 1)$. Work out the coordinates of A . [4 marks]

Answer (.....,)

- D4** The line L has equation $x + 2y = 16$. L meets the y -axis at P and the x -axis at Q . R is the point on PQ with $PR : RQ = 1 : 3$. The line through R perpendicular to L meets the x -axis at S . Work out the coordinates of S . You must show your working. [5 marks]



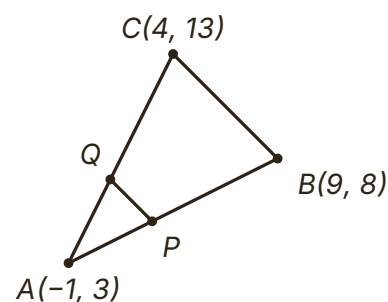
Answer (.....,)

Not drawn accurately

- D5** Triangle ABC has vertices $A(-1, 3)$, $B(9, 8)$ and $C(4, 13)$. P is on AB with $AP : PB = 2 : 3$. Q is on AC with $AQ : QC = 2 : 3$.

- (a) Work out the coordinates of P and of Q . [2 marks]

Answer $P(\dots, \dots)$ $Q(\dots, \dots)$



Not drawn accurately

- (b) Show that PQ is parallel to BC . [2 marks]

- (c) The area of triangle ABC is 37.5. Work out the area of the quadrilateral $PBCQ$. [3 marks]

Answer

Total for D5: 7 marks

- D6** P is $(1, 2)$ and Q is $(9, 14)$. R lies on PQ and on the line $y = 11$. $PR : RQ = k : 1$. [4 marks]
Work out the value of k and the coordinates of R .

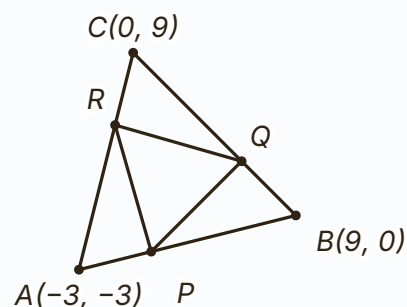
Answer $k = \dots\dots$ $R(\dots\dots, \dots\dots)$

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Triangle ABC has vertices $A(-3, -3)$, $B(9, 0)$ and $C(0, 9)$. P is on AB with $AP : PB = 1 : 2$, Q is on BC with $BQ : QC = 1 : 2$ and R is on CA with $CR : RA = 1 : 2$. Show that the area of triangle PQR is one third of the area of triangle ABC . [6 marks]



Not drawn accurately

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Find P , Q and R first. Then find both areas: put each triangle in a rectangle and take off the right-angled triangles round it.
- Hint 2** P is $\frac{1}{3}$ of the way from A to B : $(1, -2)$. In the same way Q is $(6, 3)$ and R is $(-1, 5)$.
- Hint 3** ABC sits in the 12 by 12 square from $(-3, -3)$: $144 - 18 - 40.5 - 18 = 67.5$. PQR sits in the 7 by 7 square from $(-1, -2)$: $49 - 12.5 - 7 - 7 = 22.5$. And $22.5 = \frac{1}{3} \times 67.5$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can find the point that divides a line segment in a given ratio. ☐ ☐ ☐
- I can find a point beyond the end of a line, or the far end, from a ratio. ☐ ☐ ☐
- I can work out the ratio in which a point divides a line from its coordinates. ☐ ☐ ☐
- I can use a dividing point in a longer problem, with lines and areas. ☐ ☐ ☐