



Equations of straight lines and their intersections

About 85 minutes No calculator 71 marks

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Work out the gradient of the line $5x + 2y = 9$.

Answer

A2 Needed today [2 marks]

Solve the simultaneous equations $3x + y = 11$ and $x - 2y = 6$.

Answer $x = \dots\dots$ $y = \dots\dots$

- A3** Last lesson [2 marks]

A is $(-3, 6)$ and B is $(9, 1)$. Work out the length of AB .

Answer

A4 From chapter 4 [2 marks]

Solve $x^2 - 3x - 28 < 0$.

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Work out the equation of the line that is parallel to $5x - 2y + 3 = 0$ and passes through $(2, -4)$. Give your answer in the form $ax + by + c = 0$.

Hint Parallel lines have the same gradient.
Rearrange to $y = \frac{5}{2}x + \frac{3}{2}$, or keep $5x - 2y$ and change only the number.

- B1** [3 marks]
- Work out the equation of the line that is parallel to $3x + 4y - 7 = 0$ and passes through $(-1, 5)$. Give your answer in the form $ax + by + c = 0$.

Answer = 0

EXAMPLE 2

Work out the equation of the line that is perpendicular to $3x - 4y = 8$ and passes through $(6, -1)$. Give your answer in the form $ax + by = c$, where a , b and c are integers.

Hint The given line has gradient $\frac{3}{4}$. Perpendicular gradients multiply to -1 .

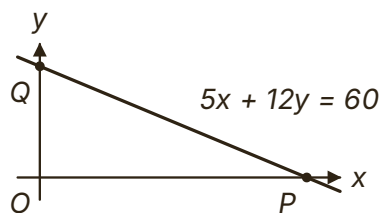
EXAMPLE 3

A is $(-2, 3)$ and B is $(6, 7)$. Work out the equation of the perpendicular bisector of AB . Give your answer in the form $y = mx + c$.

Hint The perpendicular bisector goes through the midpoint of AB at right angles to AB .

EXAMPLE 4

The line $5x + 12y = 60$ meets the x -axis at P and the y -axis at Q . Work out the area of triangle OPQ , the length PQ , and the shortest distance from O to the line.



Not drawn accurately

Hint Put $y = 0$, then $x = 0$. The shortest distance d is the height of the triangle with base PQ : area $= \frac{1}{2} \times PQ \times d$.

B2 Work out the equation of the line that is perpendicular to $2x + 5y = 9$ and passes through $(-4, 3)$. Give your answer in the form $ax + by = c$, where a , b and c are integers. [3 marks]

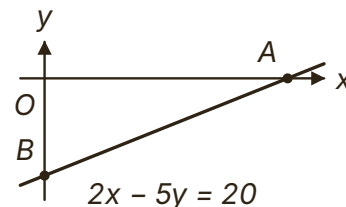
Answer

B3 P is $(-3, -4)$ and Q is $(5, 2)$. [4 marks]

Work out the equation of the perpendicular bisector of PQ . Give your answer in the form $ax + by = c$, where a , b and c are integers.

Answer

B4 The line $2x - 5y = 20$ meets the x -axis at A and the y -axis at B . Work out the area of triangle OAB , and the equation of the line through O that is perpendicular to AB . [4 marks]

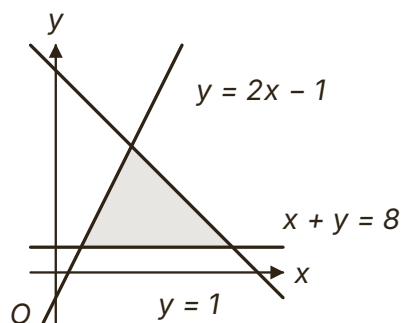


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Answer area = line:

EXAMPLE 5

The lines $y = 2x - 1$, $x + y = 8$ and $y = 1$ enclose a triangle. Work out the coordinates of its vertices and its area.

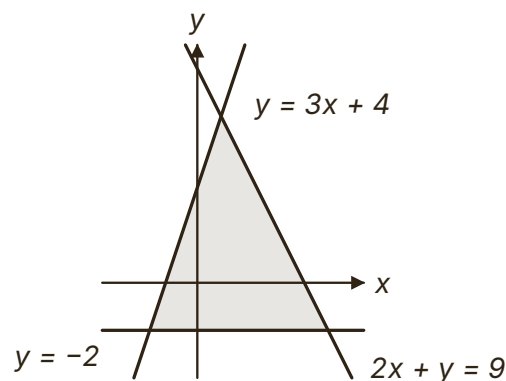


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Hint Each vertex is where two of the lines meet: solve each pair simultaneously.

B5 [5 marks]

The lines $y = 3x + 4$, $2x + y = 9$ and $y = -2$ enclose a triangle. Work out the coordinates of its vertices and its area.



Not drawn accurately

Answer area =

c Practice

Level 2

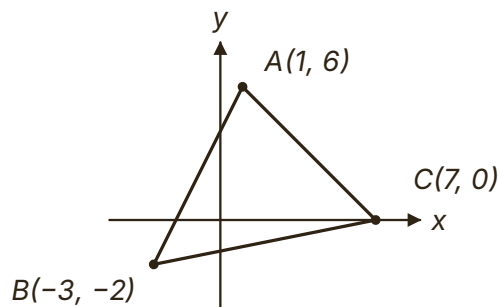
- C1** For each pair, state whether the lines are parallel, perpendicular or neither. Show how you decide. (a) $y = 3x - 2$ and $6x - 2y = 5$ (b) $2x + 5y = 1$ and $5x - 2y = 4$ (c) $x + 4y = 3$ and $4x + y = 3$ [3 marks]

- C2** The lines $3x + ky = 4$ and $4x - 6y = 1$ are perpendicular. Work out the value of k . [3 marks]

Answer $k = \dots\dots$

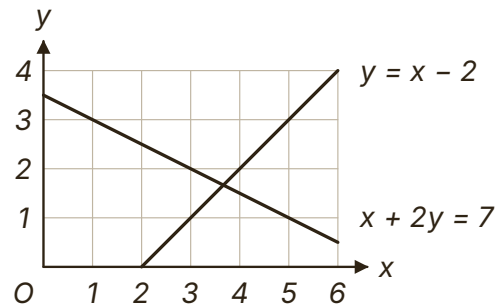
Answer

- C3** Triangle ABC has vertices $A(1, 6)$, $B(-3, -2)$ and $C(7, 0)$. A median joins a vertex to the midpoint of the opposite side. Work out the equation of the median from A . [3 marks]



Not drawn accurately

- C4** The graph shows the lines $x + 2y = 7$ and $y = x - 2$. (a) Use the graph to estimate the solution of the simultaneous equations $x + 2y = 7$ and $y = x - 2$. (b) Solve the equations algebraically to find the exact solution. [3 marks]



Answer

Answer (a) $(\dots, \dots)$ (b) $x = \dots$ $y = \dots$

D Exam-style questions

AQA exam style

- D1** Circle the gradient of a line that is perpendicular to $4x - 3y = 12$. [1 mark]
 $-\frac{4}{3}$ $-\frac{3}{4}$ $\frac{3}{4}$ $\frac{4}{3}$

Answer

- D2** Ella wants the line through $(4, -2)$ that is perpendicular to $y = 2x + 5$. She writes $y = -2x + 6$. [3 marks]

Explain Ella's mistake. Work out the correct equation.

Answer $y = \dots$

- D3** Taxi firm A charges £3.20 plus £1.40 per mile. Taxi firm B charges £5 plus £1.10 per mile. The cost of a journey of d miles is C pounds. [1 mark]
 (a) Write down an equation for C in terms of d for each firm.

Answer

- (b) Work out the length of journey for which the two firms charge the same, and that cost. You must show your working. [3 marks]

Answer miles, £

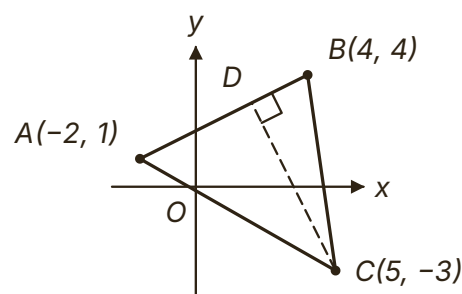
Total for D3: 4 marks

- D4** A is $(-2, 1)$, B is $(4, 4)$ and C is $(5, -3)$. The line through C perpendicular to AB meets AB at D .

- (a) Work out the equation of AB . Give your answer in the form $y = mx + c$. [2 marks]

Answer $y = \dots\dots\dots$

- (b) Work out the coordinates of D . [4 marks]



Not drawn accurately

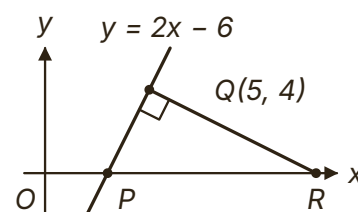
Answer $(\dots\dots, \dots\dots)$

- (c) Hence work out the area of triangle ABC . [3 marks]

Answer

Total for D4: 9 marks

- D5** Triangle PQR is bounded by the x -axis, the line $y = 2x - 6$ and a line through $Q(5, 4)$. P and R are on the x -axis. Angle $PQR = 90^\circ$. Work out the area of triangle PQR . You must show your working. [5 marks]



Not drawn accurately

Answer

D6 The lines $y = 2x - 3$ and $x + 3y = 5$ meet at the point P .

(a) Show that the line $4x - 3y = 5$ also passes through P .

[3 marks]

(b) Hence work out the equation of the line through P that is parallel to $4x + y = 1$. Give your answer in the form $ax + by = c$.

[2 marks]

Answer

Total for D6: 5 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A straight line passes through the point $(4, 3)$. It meets the positive x -axis at A and the positive y -axis at B . The area of triangle OAB is 24. Work out the equation of the line. [5 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

E1 Hint 1 Call the intercepts $A(a, 0)$ and $B(0, b)$. Write down two equations: one for the area, one saying $(4, 3)$ is on the line.

Hint 2 The line is $\frac{x}{a} + \frac{y}{b} = 1$, so $\frac{4}{a} + \frac{3}{b} = 1$, and $\frac{1}{2}ab = 24$.

Hint 3 Multiply by ab : $4b + 3a = ab = 48$, so $a = \frac{48-4b}{3}$. Then $b(48 - 4b) = 144$, $b^2 - 12b + 36 = 0$, $b = 6$ and $a = 8$. The line is $3x + 4y = 24$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write the equation of a line parallel or perpendicular to a given line through a point. ☐ ☐ ☐

I can find a perpendicular bisector and the foot of a perpendicular. ☐ ☐ ☐

I can find where lines meet the axes and each other by solving simultaneously. ☐ ☐ ☐

I can work out areas of triangles made by lines. ☐ ☐ ☐