

Expanding brackets and the binomial expansion

About 80 minutes No calculator 70 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-expanding-brackets-and-the-binomial-expansion/

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier lessons.

- A1** Needed today [2 marks]

Expand and simplify $(3x - 4)(2x + 5)$.

A2 Needed today [2 marks]

Simplify $(-2y)^3 \times (3y)^2$.

Answer

Answer

- A3** Last lesson [2 marks]

Solve $\frac{x+3}{4} - \frac{x-1}{6} = 2$.

A4 From chapter 1 [2 marks]

$a : b = 3 : 5$. Work out $(2a + b) : (b - a)$ in its simplest form.

Answer

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Expand and simplify $(x + 3)(2x - 1)(x - 4)$.

Hint Multiply two of the brackets first and simplify. Then multiply every term of that answer by each term of the last bracket.

- B1** Expand and simplify fully $(x - 3)(2x + 5)(3x - 1)$. [3 marks]

Answer

EXAMPLE 2

Expand $(3a - 2b)^3$.

Hint Row 3 of Pascal's triangle is 1, 3, 3, 1. Put $3a$ and $-2b$ into $(p + q)^3 = p^3 + 3p^2q + 3pq^2 + q^3$, with brackets round each.

- B2** Expand $(2 + 5w)^4$. [3 marks]

Answer

EXAMPLE 3

Work out the first three terms, in ascending powers of x , in the expansion of $(3 + 2x)^6$.

Hint Ascending powers start with x^0 . The first three numbers in row 6 are 1, 6, 15.

B3 Work out the first three terms, [3 marks]
in descending powers of x , in
the expansion of $(2x - 1)^7$.

Answer

EXAMPLE 4

Work out the coefficient of x^3 in the expansion of $(5 - 2x)^4$.

Hint The x^3 term uses $(-2x)^3$ and 5^1 . Its number from row 4 (1, 4, 6, 4, 1) is the fourth one.

B4 Work out the coefficient of x^3 [3 marks]
in the expansion of $(1 + 3x)^7$.

Answer

EXAMPLE 5

Work out the term independent of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^6$.

Hint A term is (number) $\times (x^2)^{6-r} \times \left(-\frac{2}{x}\right)^r$. The power of x is $12 - 2r - r$. Make it 0.

B5 Work out the term [3 marks]
independent of x in the
expansion of $\left(3x - \frac{1}{x^2}\right)^6$.

Answer

c Practice

Level 2

C1 Expand and simplify $(3y - 1)^3$. [3 marks]
Square the bracket first.

Answer

C2 Simplify [3 marks]
 $(x^2 + 3)(x - 2) - 2(x - 1)^2$.

Answer

C3 Expand and simplify

(a) $(5 - 2m)(3m + 4)$ [2 marks]

Answer

(b) $(2p - 3q)^2$ [2 marks]

Answer

Total for C3: 4 marks

C5 Pascal's triangle starts with row 0, which is 1. Row 1 is 1, 1.

(a) Write down row 7. [1 mark]

Answer

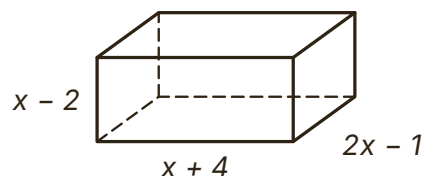
(b) Write down the third number in row 11. [1 mark]

Answer

Total for C5: 2 marks

C4 Multiply $(x^3 - 3x + 2)$ by $(x^2 + 2x - 1)$. Simplify your answer. [3 marks]

Answer

C6 The cuboid is $(x + 4)$ cm long, $(2x - 1)$ cm wide and $(x - 2)$ cm high. Give each answer expanded and simplified.

Not drawn accurately

(a) Work out the volume of the cuboid. [3 marks]

Answer cm^3

(b) Work out the total surface area of the cuboid. [3 marks]

Answer cm^2

Total for C6: 6 marks

- D1** Ella says, "The coefficient of x^2 in the expansion of $(3x + 2)^4$ is $6 \times 3 \times 2 = 36$." [3 marks]
Explain Ella's mistake. Work out the correct coefficient.

Answer

- D2** The coefficient of x^2 in the expansion of $(2 + ax)^6$ is 960.
(a) Work out the two possible values of a . [3 marks]

Answer $a = \dots\dots$ or $a = \dots\dots$

- (b) a is positive. Work out the coefficient of x^3 in the expansion. [2 marks]

Answer

Total for D2: 5 marks

- D3** Give each expansion in ascending powers of x , simplified.
(a) Expand $(1 + 3x)^4$. [2 marks]

Answer

- (b) Hence write down the expansion of $(1 - 3x)^4$. [1 mark]

Answer

- (c) Hence simplify $(1 + 3x)^4 - (1 - 3x)^4$. [2 marks]

Answer

Total for D3: 5 marks

- D4** $(x + a)(2x - 3)(x + b) \equiv 2x^3 + px^2 - 7x + 6$, where a, b and p are integers and $a > b$. [4 marks]
Work out the values of a, b and p .

Answer $a = \dots\dots$ $b = \dots\dots$ $p = \dots\dots$

- D5** n is an integer. Show that $(2n + 1)^3 - (2n - 1)^3$ is 2 more than a multiple of 24. [3 marks]

- D6** How many terms are there in the simplified expansion of $(a + b)^9$? Circle your answer. 8 9 10 11 [1 mark]

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** $x + \frac{1}{x} = 3$. Use the expansion of $\left(x + \frac{1}{x}\right)^4$ to work out the value of $x^4 + \frac{1}{x^4}$. [5 marks]

Answer $\dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1 Hint 1** Expand $\left(x + \frac{1}{x}\right)^4$ with row 4: 1, 4, 6, 4, 1. Notice that the middle three terms contain $x^2 + \frac{1}{x^2}$.
- Hint 2** $\left(x + \frac{1}{x}\right)^4 = x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$, and squaring $x + \frac{1}{x} = 3$ gives $x^2 + 2 + \frac{1}{x^2} = 9$.
- Hint 3** So $x^2 + \frac{1}{x^2} = 7$, and $81 = x^4 + \frac{1}{x^4} + 4 \times 7 + 6$, which gives $x^4 + \frac{1}{x^4} = 47$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can expand and simplify three brackets, or a product of two longer brackets. ☐ ☐ ☐

I can use a row of Pascal's triangle to expand $(a + b)^n$. ☐ ☐ ☐

I can find one term or one coefficient without the whole expansion. ☐ ☐ ☐

I can work out an unknown from a coefficient or an identity. ☐ ☐ ☐