



Function notation, domain and range

About 75 minutes No calculator 66 marks

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks] **A2** Needed today [2 marks]
- Work out the value of $2x^2 - 3x$ when $x = -4$. Solve $-3 < 2x + 1 \leq 9$.

Answer

Answer

- A3** Last lesson [2 marks] **A4** From chapter 1 [2 marks]
- Write $x^2 + 16x + 70$ in the form $(x + a)^2 + b$. Expand and simplify $(3 + \sqrt{2})(5 - 2\sqrt{2})$.

Answer

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$f(x) = x^2 - 4x + 1$. Work out $f(-3)$, and write $f(x + 2)$ in its simplest form.

Hint Put -3 in place of every x , in brackets. For $f(x + 2)$, put $(x + 2)$ in place of every x , then expand.

- B1** $h(x) = 2x^2 + 3x - 5$. Work out $h(-2)$, and write $h(x - 1)$ in its simplest form. [3 marks]

Answer

EXAMPLE 2

$f(x) = \frac{2x + 6}{x - 1}$, $x \neq 1$. Solve $f(x) = 4$, and then solve $f(2x) = 3$.

Hint Multiply both sides by the denominator. For $f(2x)$, write $2x$ in place of every x first.

- B2** $g(x) = \frac{5x - 1}{x + 3}$, $x \neq -3$. Solve $g(x) = 2$, and then solve $g(2x) = 1$. [4 marks]

Answer

EXAMPLE 3

$f(x) = ax^2 + b$. $f(1) = 5$ and $f(3) = 21$. Work out the values of a and b .

Hint Each value of f gives an equation in a and b . Solve the two equations together.

B3 $f(x) = px^2 + qx$. $f(2) = 10$ and $f(-1) = 7$. Work out the values of p and q . [3 marks]

Answer

EXAMPLE 4

$f(x) = 9 - 2x$ with domain $-3 \leq x < 5$. Work out the range of f . Give your answer as an inequality.

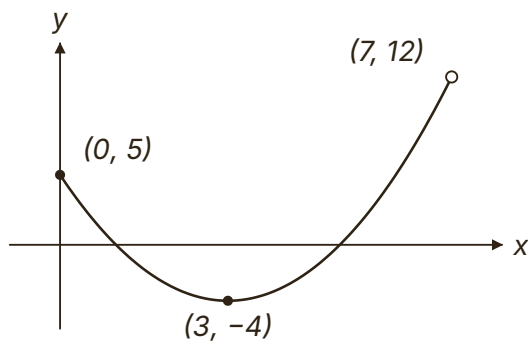
Hint Work out f at each end of the domain. The line goes down, so the left end gives the largest value. An end that is not in the domain gives a value that is not in the range.

B4 $g(x) = 10 - 4x$ with domain $-1 \leq x < 3$. Work out the range of g . Give your answer as an inequality. [2 marks]

Answer

EXAMPLE 5

$f(x) = x^2 - 6x + 5$ with domain $0 \leq x < 7$. Work out the range of f .



Not drawn accurately

Hint Complete the square to find the minimum point. Check whether it is inside the domain, then work out f at each end.

B5 $g(x) = x^2 + 4x - 1$ with domain $-5 < x \leq 1$. Work out the range of g . [3 marks]

Answer

C Practice

Level 2

C1 $f(x) = x^2 - 1$. Write $[f(x)]^2 - f(x^2)$ in its simplest form. [3 marks]

Answer

C2 $f(x) = 4x + 7$. The range of f is $-5 < f(x) \leq 19$. Work out the domain of f . [2 marks]

Answer

C3 $f(x) = 3x^2 - 18x + 31$ for all values of x . Work out the range of f . [3 marks]

Answer

C4 $g(x) = \sqrt{2x - 10}$. Write down the largest possible domain of g , and the range of g for that domain. [2 marks]

Answer

C5 $f(x) = 1 - x^3$ with domain $-1 < x \leq 2$. Work out the range of f . [2 marks]

Answer

C6 $f(x) = \frac{12}{x}$ with domain $x \geq 4$. Work out the range of f . [2 marks]

Answer

D Exam-style questions

AQA exam style

D1 Sam works out the range of $f(x) = x^2 - 2x$ with domain $-1 < x \leq 4$. Sam writes " $f(-1) = 3$ and $f(4) = 8$, so the range is $3 < f(x) \leq 8$ ". [3 marks]

Explain why Sam is wrong. Work out the correct range.

Answer

D2 This question is about three functions.

(a) $f(x) = \frac{4x+1}{5x-2}$. Which value of x cannot be in the domain of $f(x)$? Circle your answer. $-\frac{1}{4}$ 0 $\frac{2}{5}$ $\frac{5}{2}$ [1 mark]

Answer

- (b) $g(x) = 18 - 5x$ with domain $-3 < x \leq 4$. Work out the range of g . Give your answer as an inequality. [2 marks]

Answer

- (c) $h(x) = x^2 + 6x + 4$ with domain $-5 \leq x \leq 1$. Work out the range of h . [3 marks]

Answer

Total for D2: 6 marks

- D3** $f(x) = 3x + k$ with domain $2 \leq x \leq 6$, where k is a constant. The range of f is $1 \leq f(x) \leq c$. Work out the values of k and c . [3 marks]

Answer

D4 $f(x) = x^2 - x - 6$

- (a) Show that $f(x + 3) = x^2 + 5x$. [2 marks]

- (b) Hence solve $f(x + 3) = f(x)$. [2 marks]

Answer

- (c) Solve $f(2x) = 14$. [2 marks]

Answer

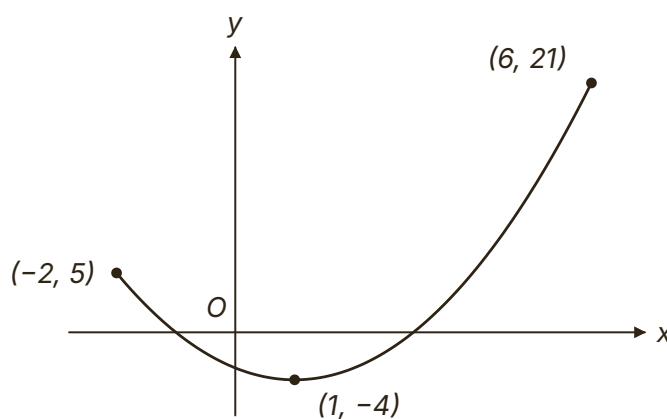
Total for D4: 6 marks

D5 The sketch shows the graph of $y = f(x)$. The curve has a minimum point at $(1, -4)$. Its end points are $(-2, 5)$ and $(6, 21)$.

- (a) Write down the domain and the range of f . [2 marks]

Answer

- (b) The equation $f(x) = k$ has exactly two solutions. Work out the range of values of k . [2 marks]



Not drawn accurately

Answer

- (c) $f(x) = x^2 + px + q$. Work out the values of p and q . [2 marks]

Answer

Total for D5: 6 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** $f(x) = x^2 - 4x + 7$ with domain $a \leq x \leq 5$, where a is a constant. The range of f is $3 \leq f(x) \leq 12$. Work out all the possible values of a . [5 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Complete the square and sketch the curve. Which value of x gives the smallest value 3, and which gives 12?
- Hint 2** $f(x) = (x - 2)^2 + 3$. The smallest value 3 is at $x = 2$, so $x = 2$ must be in the domain: $a \leq 2$.
- Hint 3** $f(5) = 12$, so the largest value is 12 as long as $f(a) \leq 12$. $(a - 2)^2 + 3 \leq 12$ gives $(a - 2)^2 \leq 9$, so $-1 \leq a \leq 5$. With $a \leq 2$, the answer is $-1 \leq a \leq 2$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can work out $f(a)$ and write f of an expression in its simplest form.



I can solve equations written in function notation, including with fractions.



I can find the range of a function from its domain, checking for a turning point inside it.



I can say which values cannot be in a domain, and find a domain from a range.

