



Functions with up to three parts to their domains

About 85 minutes No calculator 59 marks

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks]

Work out the equation of the straight line through (2, 7) and (6, -1).

A2 Last lesson [2 marks]

The curve $y = ab^x$ passes through (0, 4) and (2, 36), where a and b are positive. Work out a and b .

Answer

Answer

- A3** From chapter 3 [2 marks]

$f(x) = \frac{x+4}{5}$. Work out an expression for $f^{-1}(x)$.

A4 Needed today [2 marks]

$g(x) = 7 - 2x$ for $-1 \leq x \leq 3$. Work out the range of g .

Answer

Answer

B Example, then your turn

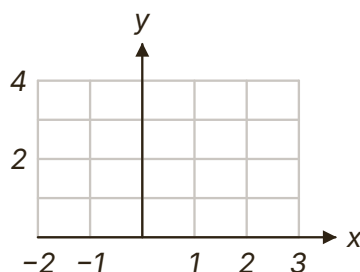
Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Draw the graph of $y = f(x)$ on the grid, where

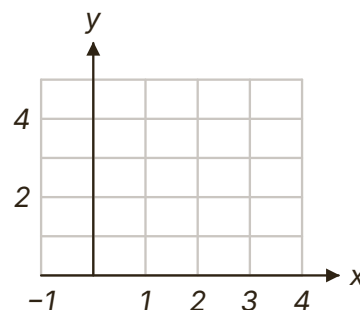
$$f(x) = \begin{cases} 2x + 4, & -2 \leq x < 0 \\ 4, & 0 \leq x \leq 3 \end{cases}$$



Hint Work out f at each end of each part. The first part is a straight line; the second is a horizontal line.

- B1** Draw the graph of $y = f(x)$ on the grid, where [3 marks]

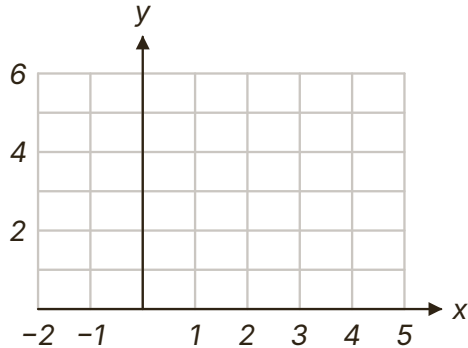
$$f(x) = \begin{cases} 5, & -1 \leq x < 2 \\ 9 - 2x, & 2 \leq x \leq 4 \end{cases}$$



EXAMPLE 2

Draw the graph of $y = f(x)$ on the grid, where

$$f(x) = \begin{cases} 3, & -2 \leq x < 0 \\ (x-1)^2 + 2, & 0 \leq x < 3 \\ 12 - 2x, & 3 \leq x \leq 5 \end{cases}$$



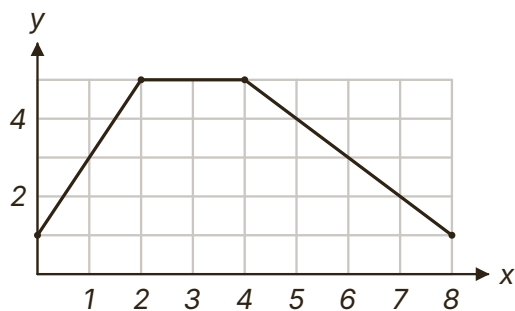
Hint For the curved part make a table for $x = 0, 1, 2, 3$ and join the points with a smooth curve. Check where each part starts and ends.

EXAMPLE 3

The graph of $y = f(x)$ is drawn for $0 \leq x \leq 8$.

(a) Define $f(x)$, stating the domain of each part.

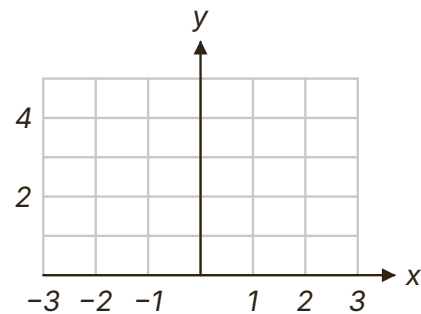
(b) State the range of f .



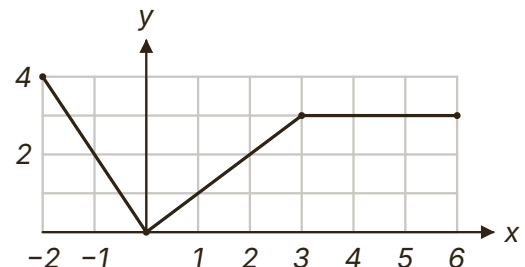
Hint Find the gradient and the y -intercept of each straight piece. Each value of x belongs to one part only, so use $<$ at one end and $\leq$ at the other.

B2 Draw the graph of $y = f(x)$ on the grid, where [3 marks]

$$f(x) = \begin{cases} x + 5, & -3 \leq x < -1 \\ 5 - x^2, & -1 \leq x < 1 \\ 6 - 2x, & 1 \leq x \leq 3 \end{cases}$$



B3 The graph of $y = f(x)$ is drawn for $-2 \leq x \leq 6$.



(a) Define $f(x)$, stating the domain of each part. [3 marks]

(b) State the range of f . [1 mark]

Answer

Total for B3: 4 marks

EXAMPLE 4

$$f(x) = \begin{cases} 3x + 2, & x < 0 \\ x^2 - 6x + 2, & 0 \leq x \leq 6 \\ 3x - 16, & x > 6 \end{cases}$$

Solve $f(x) = -3$.

Hint Solve $f(x) = -3$ for each part on its own. Then keep only the answers that lie in that part's domain.

$$\mathbf{B4} \quad f(x) = \begin{cases} 2x + 1, & x < 0 \\ x^2 - 4x + 1, & 0 \leq x \leq 5 \\ 21 - 3x, & x > 5 \end{cases} \quad [4 \text{ marks}]$$

Solve $f(x) = -2$.

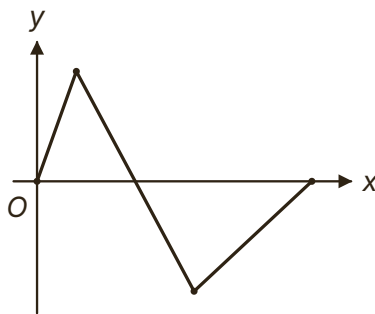
Answer $x = \dots\dots\dots$

c Practice

Level 2

$$\mathbf{C1} \quad f(x) = \begin{cases} 3x, & 0 \leq x < 1 \\ 5 - 2x, & 1 \leq x < 4 \\ x - 7, & 4 \leq x \leq 7 \end{cases}$$

The sketch shows $y = f(x)$. The graph meets the x -axis at $x = 0$, $x = a$ and $x = 7$.



Not drawn accurately

(a) Work out the value of a . [1 mark]

Answer $\dots\dots$ km/h

Answer $a = \dots\dots$

(b) Work out the ratio of the area above the x -axis to the area below it, between the graph and the x -axis. Give your answer in its simplest form. [3 marks]

Answer $\dots\dots\dots$

Total for C1: 4 marks

C2 A walker sets off from home. Her distance from home, d km, after t hours is

$$d = \begin{cases} 4t, & 0 \leq t < 1.5 \\ 6, & 1.5 \leq t < 2 \\ 12 - 3t, & 2 \leq t \leq 4 \end{cases}$$

(a) Describe what the walker does between $t = 1.5$ and $t = 2$. [1 mark]

(b) Work out her speed on the way home. [1 mark]

(c) Work out the two times when she is 3 km from home. [2 marks]

Answer $t = \dots\dots$ and $t = \dots\dots$

Total for C2: 4 marks

- C3** $f(x) = \begin{cases} x^2 + k, & 0 \leq x < 3 \\ 2x + 5, & 3 \leq x \leq 6 \end{cases}$ [2 marks] **C4** $f(x) = \begin{cases} 2 - x, & -3 \leq x < 0 \\ x^2 + 2, & 0 \leq x < 2 \\ 10 - 2x, & 2 \leq x \leq 4 \end{cases}$ [2 marks]
- The two parts of the graph of $y = f(x)$ join at $x = 3$. Work out the value of k .
- Work out the range of f .

Answer

Answer $k = \dots\dots$

D Exam-style questions

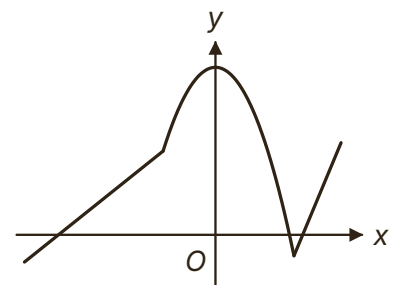
AQA exam style

- D1** $f(x) = \begin{cases} x^2, & x < 4 \\ 15 - x, & x \geq 4 \end{cases}$ [1 mark]
- Circle the value of $f(4)$.
- 7 11 12 16

Answer

- D2** A function f is given by [5 marks]
- $$f(x) = \begin{cases} x + 6, & x < -2 \\ 8 - x^2, & -2 \leq x \leq 3 \\ 3x - 10, & x > 3 \end{cases}$$

A sketch of $y = f(x)$ is shown. Work out all the values of x for which $f(x) = 0$. Give your answers in exact form. You must show your working.



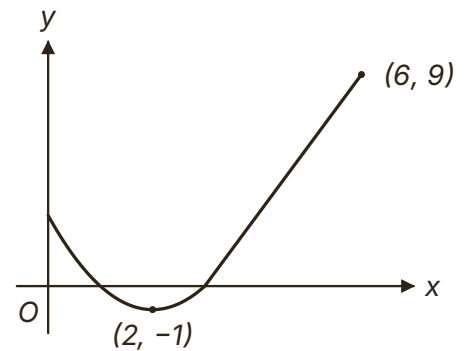
Answer $x = \dots\dots\dots$

Not drawn accurately

D3 $f(x) = \begin{cases} (x-a)^2 + b, & 0 \leq x < 3 \\ cx + d, & 3 \leq x \leq 6 \end{cases}$ [4 marks]

a , b , c and d are constants. The sketch shows $y = f(x)$. The curve has its minimum point at $(2, -1)$, the graph ends at $(6, 9)$, and the two parts join at $x = 3$. Work out the values of a , b , c and d .

Answer $a = \dots\dots$ $b = \dots\dots$ $c = \dots\dots$ $d = \dots\dots$



Not drawn accurately

D4

$$f(x) = \begin{cases} 2x + 8, & -4 \leq x < -1 \\ x^2 + 5, & -1 \leq x < 2 \\ 13 - 2x, & 2 \leq x \leq 5 \end{cases}$$

Priya says, "The equation $f(x) = 4$ has three solutions, one from each part."

Priya is wrong.

(a) Show that $f(x) = 4$ has exactly two solutions, and work them out. [3 marks]

Answer $x = \dots\dots$ and $x = \dots\dots$

(b) Work out the range of f . [2 marks]

Answer

Total for D4: 5 marks

D5 A train starts from rest and speeds up at a steady rate for 40 seconds until its speed is 20 m/s. It then travels at 20 m/s for T seconds. It then slows down at a steady rate for 50 seconds until it stops. The train travels 4500 m in total.

(a) Work out the value of T . [3 marks]

Answer $T = \dots\dots$

- (b) The speed of the train is v m/s after t seconds. Write v as a function of t , stating the domain of each part.

[3 marks]

Total for D5: 6 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1
$$f(x) = \begin{cases} x + 3, & -3 \leq x < 0 \\ x^2 - 4x + 3, & 0 \leq x < 4 \\ 11 - 2x, & 4 \leq x \leq 6 \end{cases}$$
 [4 marks]

Work out the values of k for which the equation $f(x) = k$ has exactly three solutions.

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Sketch $y = f(x)$, then imagine the horizontal line $y = k$ moving up and down.
- Hint 2** The middle part is $(x - 2)^2 - 1$: its lowest point is $(2, -1)$. Each part runs from $y = 3$ (at one end) down to its lowest value.
- Hint 3** For $k < -1$ there are no solutions; $k = -1$ gives two; for $-1 < k < 0$ the curve gives two and the last part one; for $0 \leq k < 3$ the first part adds one more, making four. So $-1 < k < 0$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can draw the graph of a function defined in two or three parts. ☐ ☐ ☐

I can define a function from its graph, giving the domain of each part, and state its range. ☐ ☐ ☐

I can solve $f(x) = k$, keeping only the solutions that lie in their own part. ☐ ☐ ☐

I can use speed-time and distance-time functions in context. ☐ ☐ ☐