



Geometric proof

About 75 minutes No calculator 55 marks

Date _____

A Do Now

The first is what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

In triangle ABC , angle $A = (2x + 10)^\circ$ and angle $B = (3x - 5)^\circ$. The exterior angle at C is 105° . Work out the value of x .

A2 Last lesson [2 marks]

$ABCD$ is a cyclic quadrilateral. Angle $A = (2x + 16)^\circ$ and angle $C = (x + 38)^\circ$. Work out the size of angle A .

Answer _____

Answer _____

- A3** From chapter 4 [2 marks]

Show that $(n + 3)^2 - (n - 3)^2$ is a multiple of 12 for every integer n .

A4 From chapter 5 [2 marks]

Work out the gradient of a line perpendicular to the line through $(1, 4)$ and $(5, 12)$.

Answer _____

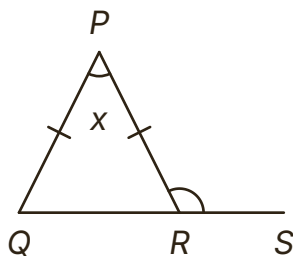
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it. Give a reason for every step.

EXAMPLE 1

$PQ = PR$. QRS is a straight line. Angle $QPR = x$. Prove that angle $PRS = 90^\circ + \frac{1}{2}x$.

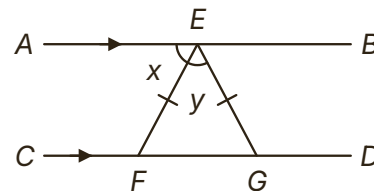


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Hint Start with the two equal angles of the isosceles triangle. Then use the straight line.

B1 [3 marks]

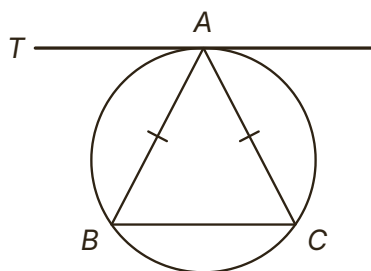
AB is parallel to CD . $EF = EG$. Angle $AEF = x$ and angle $FEG = y$. Prove that $y = 180^\circ - 2x$.



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EXAMPLE 2

TA is a tangent to the circle at A . $AB = AC$.
Prove that TA is parallel to BC .

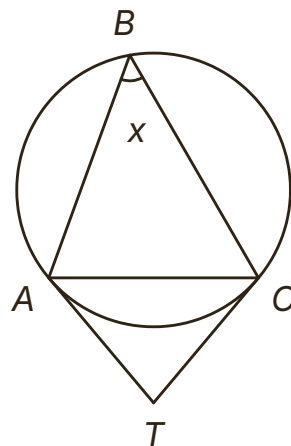


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Hint Use the alternate segment theorem for the angle between TA and AB . Then look for alternate angles.

B2 [3 marks]

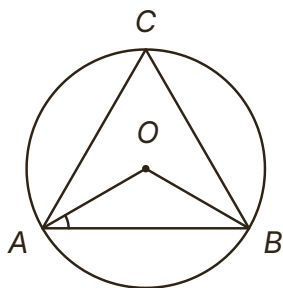
TA and TC are tangents to the circle at A and C . B is on the major arc AC . Angle $ABC = x$. Prove that angle $ATC = 180^\circ - 2x$.



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EXAMPLE 3

O is the centre. C is on the major arc AB .
Angle $OAB = x$. Prove that angle $ACB = 90^\circ - x$.

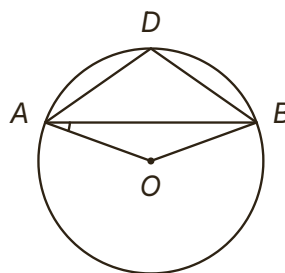


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Hint Triangle OAB has two radii as sides. Find angle AOB , then use the angle at the centre.

B3 [4 marks]

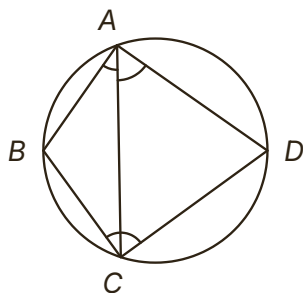
O is the centre. D is on the minor arc AB . Angle $OAB = x$. Prove that angle $ADB = 90^\circ + x$.



Not drawn accurately

EXAMPLE 4

A, B, C and D are points on a circle. Angle $BAC = 2x$, angle $CAD = 3x$ and angle $BCD = 5x$. Prove that BD is a diameter of the circle.

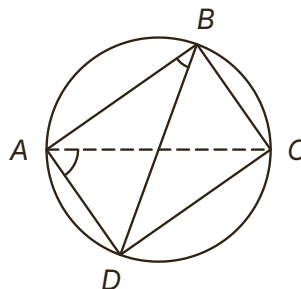


Not drawn accurately

Hint Opposite angles of a cyclic quadrilateral add up to 180° . A chord is a diameter if the angle it makes at the circumference is 90° .

B4 A, B, C and D are points on a circle. [3 marks]

Angle $CAD = x$ and angle $ABD = 90^\circ - x$. Prove that AC is a diameter of the circle.

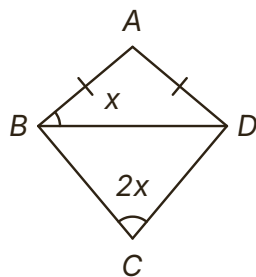


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c Practice

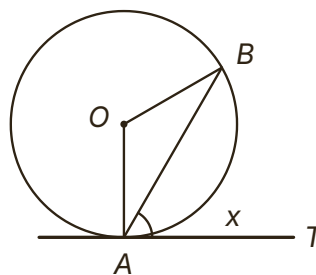
Level 2

- C1** $ABCD$ is a quadrilateral. [3 marks]
 $AB = AD$. Angle $ABD = x$ and angle $BCD = 2x$. Prove that $ABCD$ is a cyclic quadrilateral.



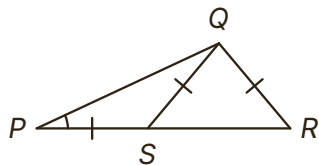
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- C2** O is the centre. TA is a tangent [3 marks]
to the circle at A . Angle $TAB = x$. Prove that angle $OBA = 90^\circ - x$.



Not drawn accurately

- C3** S is a point on the side PR of triangle PQR . $PS = SQ = QR$.
Angle $QPS = x$. Prove that
angle $PQR = 180^\circ - 3x$. [3 marks]



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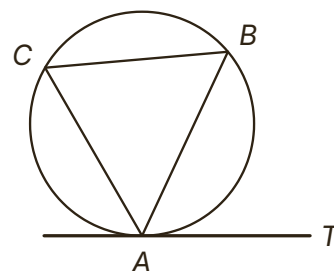
D Exam-style questions

AQA exam style

- D1** TA is a tangent to the circle at A . B and C are points on the circle. Ben says, "Angle $TAB = \text{angle } ABC$, by the alternate segment theorem." [2 marks]

Ben is wrong. Which angle in the diagram is equal to angle TAB ? Give the reason.

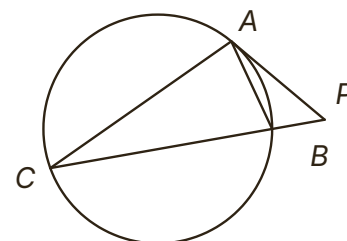
Answer angle



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- D2** A , B and C are points on a circle. CBP is a straight line. Angle $ACB = x$, angle $ABC = 3x$ and angle $APB = 2x$.

- (a) Prove that PA is a tangent to the circle. [4 marks]



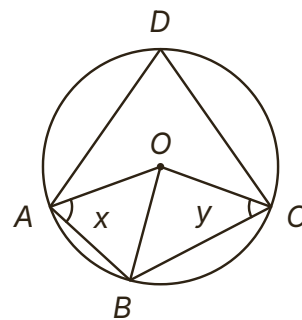
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- (b) Angle $BAC = 44^\circ$. Work out angle APB . [2 marks]

Answer $^\circ$

Total for D2: 6 marks

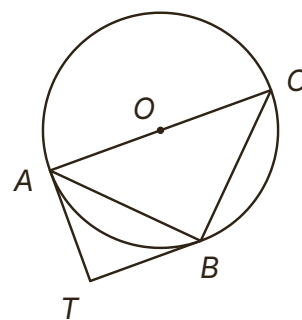
- D3** O is the centre. B is on the minor arc AC and D is on the major arc AC . Angle $OAB = x$ and angle $OCB = y$. Prove that angle $ADC = 180^\circ - x - y$. [4 marks]



Not drawn accurately

- D4** O is the centre. TA and TB are tangents to the circle at A and B . AC is a diameter.

- (a) Prove that angle $ATB = 2 \times$ angle BAC . [4 marks]



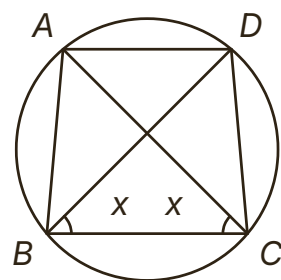
- (b) Angle $ATB = 64^\circ$. Hence work out angle ACB . [2 marks]

Not drawn accurately

Answer $^\circ$

Total for D4: 6 marks

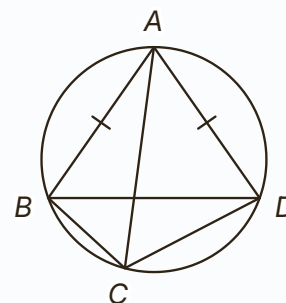
- D5** A, B, C and D are points on a circle. Angle $ACB = x$ and angle $DBC = x$. Prove that AD is parallel to BC . [3 marks]



Not drawn accurately

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** $ABCD$ is a cyclic quadrilateral with $AB = AD$. Prove that CA bisects angle BCD . [4 marks]



Not drawn accurately

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Draw the diagonal BD . Which two angles does $AB = AD$ make equal?
- Hint 2** Angle $ABD = \text{angle } ADB$ (isosceles triangle). Now find an angle at C equal to each of them.
- Hint 3** Angle $ACD = \text{angle } ABD$ and angle $ACB = \text{angle } ADB$ (angles in the same segment), so angle $ACB = \text{angle } ACD$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write a geometric proof with a reason for every step. ☐ ☐ ☒

I can use the circle theorems and their converses in a proof. ☐ ☐ ☒

I can use a proved result to work out an angle. ☐ ☐ ☒