



Area of a triangle, the sine and cosine rules, and problems in three dimensions

About 90 minutes Calculator allowed

122 marks

Date _____

A Do Now

The first one is what today needs. The others bring back earlier work.

A1 Needed today [2 marks]

Write down the exact value of $\sin 120^\circ$ and the exact value of $\cos 150^\circ$.

Answer $\sin 120^\circ = \dots\dots$ $\cos 150^\circ = \dots\dots$

A2 From chapter 6 [2 marks]

Solve $5 \cos x + 2 = 0$ for $0^\circ \leq x \leq 360^\circ$. Give your answers to 1 decimal place.

Answer

A3 From chapter 4 [2 marks]

Solve $x^2 - 3x - 54 = 0$.

Answer

A4 GCSE Higher [2 marks]

In triangle PQR , angle $PQR = 90^\circ$, $PQ = 6.4$ cm and $QR = 9.1$ cm. Work out the size of angle QPR . Give your answer to 1 decimal place.

Answer $\dots\dots^\circ$

Work out the distance between the points $(-1, 4)$ and $(5, -2)$. Give your answer as a surd in its simplest form.

Answer

B

Examples, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it on your own. Give lengths and angles to 3 significant figures.

EXAMPLE 1

In triangle ABC , $AB = 12$ cm, $AC = 9$ cm and angle $ABC = 34^\circ$.

Work out the two possible lengths of BC .

Hint The sine rule gives $\sin C = \frac{12 \sin 34^\circ}{9}$. The calculator gives the acute angle; 180° minus it also works, as it still leaves room in the triangle. Find angle A for each, then use the sine rule again for BC .

B1 In triangle PQR , $PQ = 14$ cm, $PR = 10$ cm and angle $PQR = 41^\circ$. [4 marks]

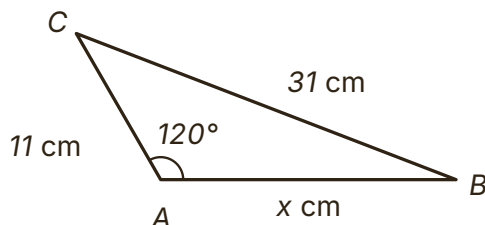
Work out the two possible lengths of QR .

Answer $QR = \dots\dots$ cm or $\dots\dots$ cm

EXAMPLE 2

In triangle ABC , angle $BAC = 120^\circ$, $AC = 11$ cm and $BC = 31$ cm. $AB = x$ cm.

Work out the value of x .



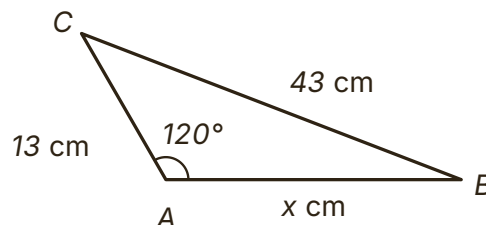
Not drawn accurately

Hint Use the cosine rule for BC , the side opposite 120° : $31^2 = x^2 + 11^2 - 2 \times 11x \cos 120^\circ$. As $\cos 120^\circ = -\frac{1}{2}$, this is $x^2 + 11x - 840 = 0$. Factorise and keep the positive root.

B2 In triangle ABC , angle $BAC = 120^\circ$, $AC = 13$ cm [4 marks]

and $BC = 43$ cm. $AB = x$ cm.

Work out the value of x .



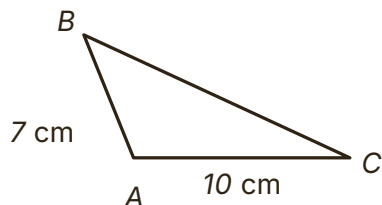
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Answer $x = \dots\dots$

EXAMPLE 3

In triangle ABC , $AB = 7$ cm and $AC = 10$ cm. The area of the triangle is 28 cm^2 and angle BAC is obtuse.

Work out the length of BC .

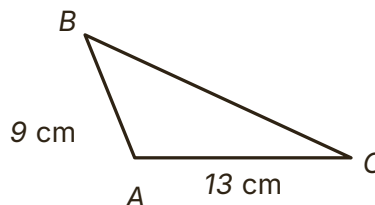


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Hint $\frac{1}{2} \times 7 \times 10 \sin A = 28$, so $\sin A = 0.8$. The obtuse angle is $180^\circ - 53.1^\circ = 126.9^\circ$. Then the cosine rule gives BC ; $\cos 126.9^\circ$ is negative, so BC^2 is more than $7^2 + 10^2$.

B3 In triangle ABC , $AB = 9$ cm [4 marks] and $AC = 13$ cm. The area of the triangle is 35.1 cm^2 and angle BAC is obtuse.

Work out the length of BC .



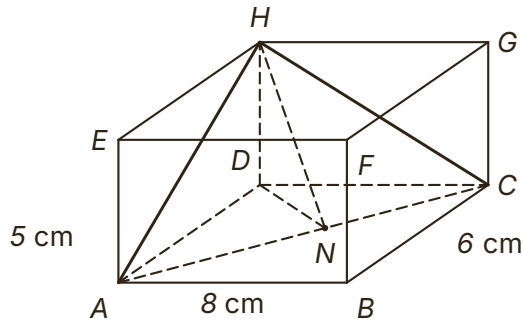
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Answer $BC = \dots\dots$ cm

EXAMPLE 4

$ABCDEFGH$ is a cuboid with $AB = 8$ cm, $BC = 6$ cm and $AE = 5$ cm. N is the point on AC such that DN is perpendicular to AC .

Work out the angle between the plane ACH and the base $ABCD$.



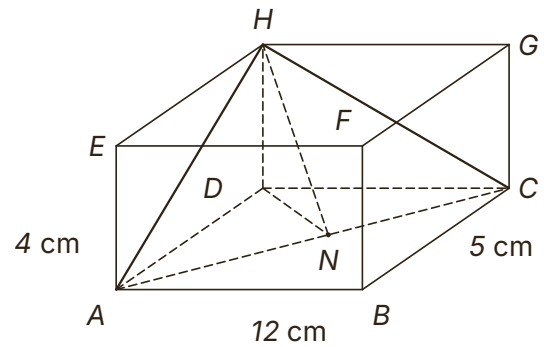
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Hint The angle is HND , as HN and DN are both perpendicular to AC . In triangle ADC , $AC = 10$ and the area is $\frac{1}{2} \times 6 \times 8 = \frac{1}{2} \times 10 \times DN$, so $DN = 4.8$. Then $\tan HND = \frac{5}{4.8}$.

B4 $ABCDEFGH$ is a cuboid [4 marks]

with $AB = 12$ cm, $BC = 5$ cm and $AE = 4$ cm. N is the point on AC such that DN is perpendicular to AC .

Work out the angle between the plane ACH and the base $ABCD$.



Not drawn accurately

Answer°

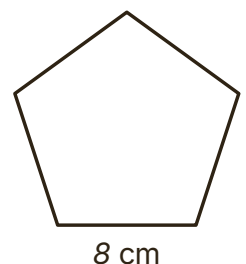
c Practice

Level 2

Each question asks for something different. Show your working. Give lengths and angles to 3 significant figures unless the question says otherwise.

C1 A regular pentagon has sides of 8 cm. Work out its area.

[4 marks]

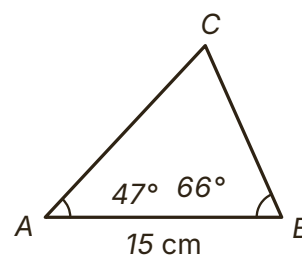


Answer cm²

- C2** In triangle ABC , $AB = 15$ cm, angle $CAB = 47^\circ$ and angle $ABC = 66^\circ$. [4 marks]

Work out the perimeter of the triangle.

Answer cm

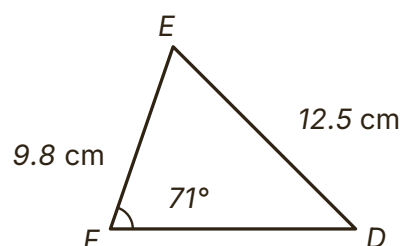


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- C3** In triangle DEF , $DE = 12.5$ cm, $EF = 9.8$ cm and angle $DFE = 71^\circ$. [3 marks]

Work out the size of angle EDF . Explain why there is only one possible answer.

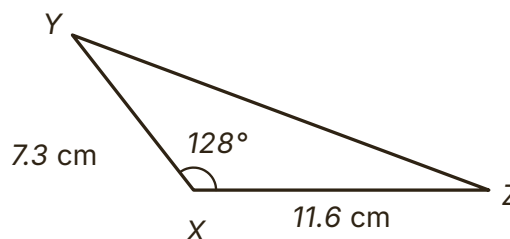
Answer angle $EDF = \dots\dots^\circ$



Not drawn accurately

- C4** In triangle XYZ , $XY = 7.3$ cm, $XZ = 11.6$ cm and angle $YXZ = 128^\circ$. Work out the length of YZ . [3 marks]

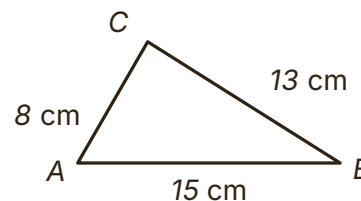
Answer $YZ = \dots\dots$ cm



Not drawn accurately

- C5** Triangle ABC has $AB = 15$ cm, $AC = 8$ cm and $BC = 13$ cm.

- (a) Show that angle $BAC = 60^\circ$. [3 marks]



(b) Work out the exact area of the triangle.

[2 marks]

Answer cm^2

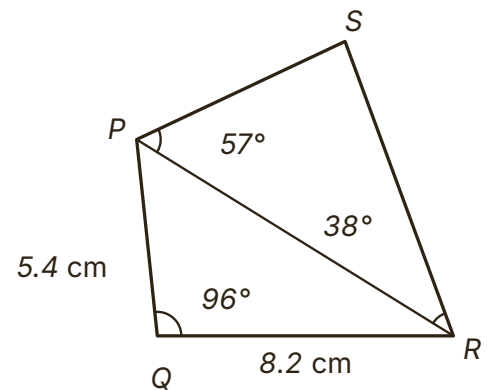
Total for C5: 5 marks

C6 $PQRS$ is a quadrilateral. $PQ = 5.4$ cm, $QR = 8.2$ cm, angle $PQR = 96^\circ$, angle $PRS = 38^\circ$ and angle $RPS = 57^\circ$.

(a) Work out the length of PR . [2 marks]

Answer $PR = \dots\dots$ cm

(b) Work out the length of RS . [2 marks]



Not drawn accurately

Answer $RS = \dots\dots$ cm

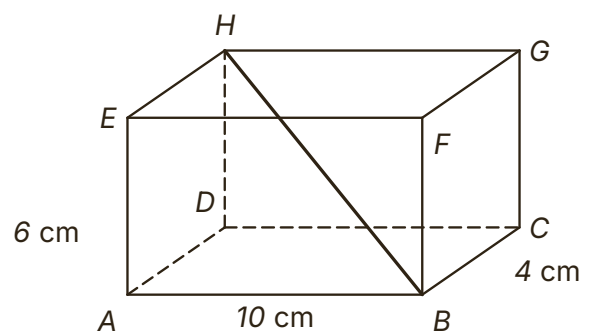
Total for C6: 4 marks

C7 $ABCDEFGH$ is a cuboid with $AB = 10$ cm, $BC = 4$ cm and $AE = 6$ cm.

(a) Work out the length of the diagonal BH . [2 marks]

Answer $BH = \dots\dots$ cm

(b) Work out the angle between BH and the base $ABCD$. [2 marks]



Not drawn accurately

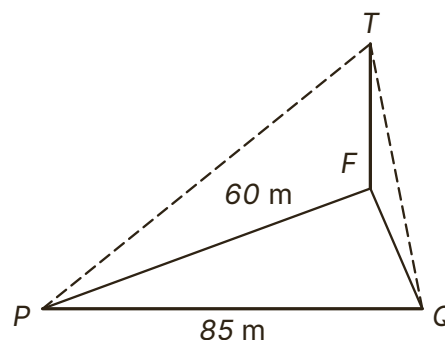
Answer $^\circ$

Total for C7: 4 marks

- C8** P, Q and F are on level ground. FT is a vertical mast. $PQ = 85$ m, $PF = 60$ m [5 marks]
and angle $FPQ = 48^\circ$. The angle of elevation of T from P is 26° .

Work out the angle of elevation of T from Q .

Answer $^\circ$

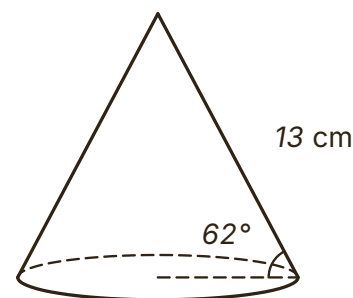


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- C9** A cone has a slant height of 13 cm. The slant edge makes an angle of 62° with the base. Work out the volume of the cone. [3 marks]

(The volume of a cone is $\frac{1}{3}\pi r^2 h$.)

Answer cm^3



Not drawn accurately

D Exam-style questions

AQA exam style

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** A triangle has sides of 6 cm and 9 cm with an angle of 150° between them. [1 mark]
Which of these is its area, in cm^2 ? Circle your answer.

13.5 27 $13.5\sqrt{3}$ $27\sqrt{3}$

Answer

- D2** Leo says, "A triangle with sides 4 cm, 6 cm and 11 cm has an obtuse angle, because 11^2 is more than $4^2 + 6^2$." [3 marks]

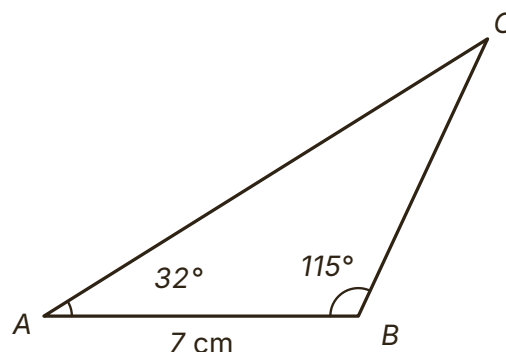
Use the cosine rule to show that Leo is wrong.

- D3** In triangle ABC , $AB = 7$ cm, angle $CAB = 32^\circ$ and angle $ABC = 115^\circ$.

- (a) Work out the length of BC . [2 marks]

Answer $BC = \dots\dots$ cm

- (b) Work out the area of the triangle. [2 marks]



Not drawn accurately

Answer $\dots\dots$ cm^2

Total for D3: 4 marks

- D4** In triangle ABC , $AC = 10$ cm and $BC = 14$ cm. The area of the triangle is 42 cm^2 .

- (a) Show that $\sin ACB = 0.6$. [2 marks]

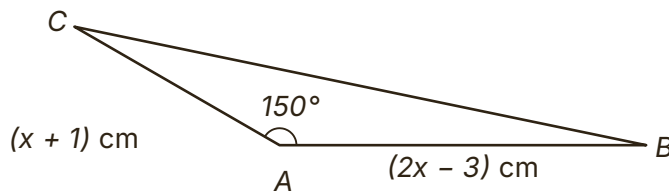
- (b) Hence work out the two possible lengths of AB . Give your answers as surds in their simplest form. [4 marks]

Answer $\dots\dots$ cm or $\dots\dots$ cm

Total for D4: 6 marks

- D5** In triangle ABC , $AC = (x + 1)$ cm, $AB = (2x - 3)$ cm and angle $BAC = 150^\circ$. The area of the triangle is 10.5 cm^2 .

- (a) Show that [3 marks]
 $2x^2 - x - 45 = 0$.



Not drawn accurately

- (b) Hence work out the length of BC .

[3 marks]

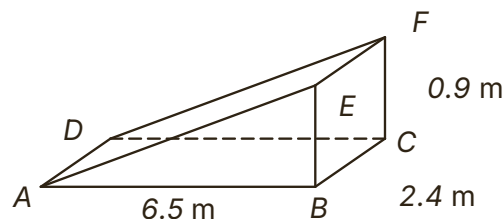
Answer $BC = \dots\dots$ cm

Total for D5: 6 marks

- D6** The diagram shows a ramp. The base $ABCD$ is a horizontal rectangle and the face $BCFE$ is vertical. $AB = 6.5$ m, $BC = 2.4$ m and $CF = 0.9$ m.

A ramp is allowed if its sloping face $ADFE$ makes an angle of less than 8° with the horizontal.

- (a) Is this ramp allowed? You must [2 marks]
show your working.



Answer $\dots\dots^\circ$ allowed? $\dots\dots$

Not drawn accurately

- (b) Work out the length of AF . [2 marks]

Answer $AF = \dots\dots$ m

- (c) Work out the angle between AF and the base $ABCD$.

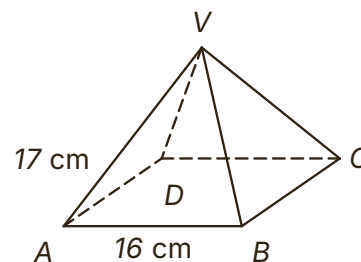
[2 marks]

Answer $\dots\dots^\circ$

Total for D6: 6 marks

- D7** $VABCD$ is a pyramid. Its base $ABCD$ is a horizontal square of side 16 cm. V is directly above the centre of the base, and each sloping edge is 17 cm long.

- (a) Work out the size of angle AVC . [3 marks]



Answer°

- (b) Hence work out the angle between VA and the base. [1 mark] Not drawn accurately

Answer°

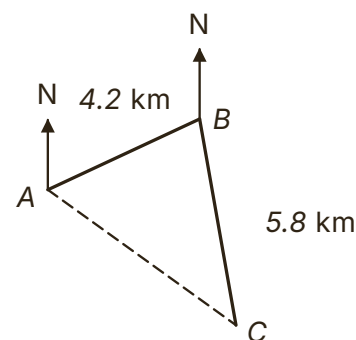
- (c) Work out the angle between the face VBC and the base. [3 marks]

Answer°

Total for D7: 7 marks

- D8** An orienteer runs 4.2 km from A to B on a bearing of 065° . She then runs 5.8 km from B to C on a bearing of 170° .

- (a) Show that angle $ABC = 75^\circ$. [2 marks]



- (b) Work out the distance AC . [2 marks]

Not drawn accurately

Answer $AC = \dots\dots$ km

- (c) Work out the bearing of A from C . [3 marks]

Answer°

(d) Work out the area of the triangle ABC .

[2 marks]

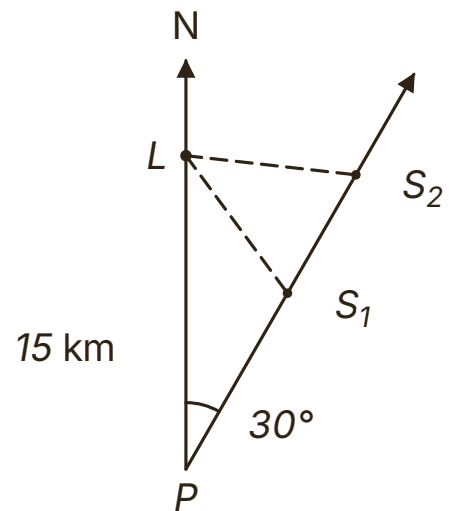
Answer km^2

Total for D8: 9 marks

- D9** A lighthouse L is 15 km due north of a port P . A ship sails from P on a bearing of 030° . It is exactly 9 km from the lighthouse at two points, S_1 and S_2 , on its course. [4 marks]

Work out the distance between S_1 and S_2 .

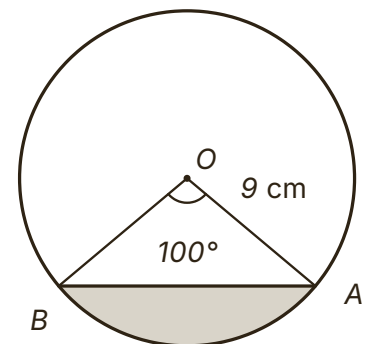
Answer km



Not drawn accurately

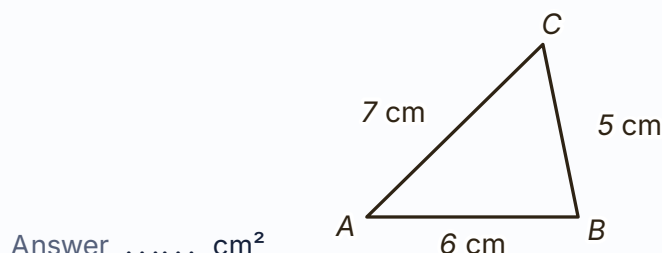
- D10** AB is a chord of a circle, centre O , radius 9 cm. Angle $AOB = 100^\circ$. [3 marks]
- Work out the area of the shaded segment.

Answer cm^2



No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

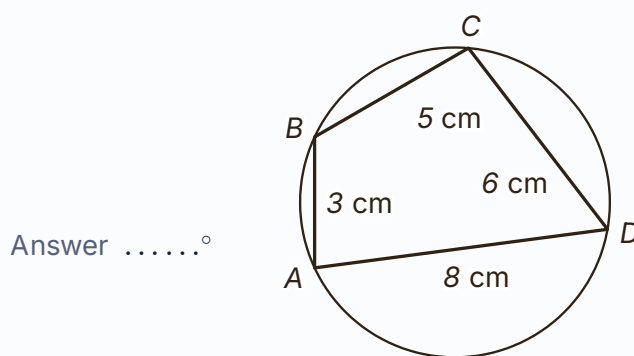
- E1** A triangle has sides of 5 cm, 6 cm and 7 cm. Work out its exact area. [4 marks]



Answer cm^2

- E2** $ABCD$ is a quadrilateral whose four vertices lie on a circle. $AB = 3$ cm, $BC = 5$ cm, $CD = 6$ cm and $DA = 8$ cm. [4 marks]

Work out the size of angle ABC .

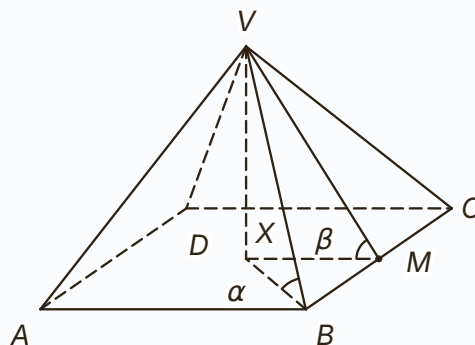


Answer $^\circ$

Not drawn accurately

- E3** $VABCD$ is a pyramid on a horizontal square base, with V directly above the centre X of the base. M is the midpoint of BC . The edge VB makes an angle α with the base and the face VBC makes an angle β with the base. [4 marks]

Show that $\tan \beta = \sqrt{2} \tan \alpha$.



F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can find the area of a triangle with $\frac{1}{2}ab\sin C$, and work back from an area to an angle or a side.	☐	☐	☐
I can use the sine rule for a side or an angle, and I know when there are two possible triangles.	☐	☐	☐
I can use the cosine rule for a side or an angle, including when it gives a quadratic, and use it with bearings.	☐	☐	☐
I can find lengths and angles in three dimensions, including the angle between a line and a plane and between two planes.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

E1 **Hint 1** Use the cosine rule for one angle, then $\frac{1}{2}ab \sin C$. Keep everything exact.

Hint 2 The angle opposite the 7 cm side: $\cos C = \frac{25 + 36 - 49}{2 \times 5 \times 6} = \frac{1}{5}$.

Hint 3 $\sin C = \sqrt{1 - \frac{1}{25}} = \frac{2\sqrt{6}}{5}$, so the area is $\frac{1}{2} \times 5 \times 6 \times \frac{2\sqrt{6}}{5} = 6\sqrt{6}$.

E2 **Hint 1** Opposite angles of a cyclic quadrilateral add to 180° , so $\cos ADC = -\cos ABC$. The diagonal AC is in two triangles.

Hint 2 Write AC^2 twice: $3^2 + 5^2 - 30 \cos B = 6^2 + 8^2 + 96 \cos B$.

Hint 3 $34 - 30 \cos B = 100 + 96 \cos B$, so $126 \cos B = -66$, $\cos B = -\frac{11}{21}$ and $B = 121.6^\circ$.

E3 **Hint 1** Call the side of the base $2a$ and the height h . Both angles are in right-angled triangles with the vertical side VX .

Hint 2 XB is half the diagonal, $a\sqrt{2}$, so $\tan \alpha = \frac{h}{a\sqrt{2}}$. $XM = a$, so $\tan \beta = \frac{h}{a}$.

Hint 3 $\tan \beta = \frac{h}{a} = \sqrt{2} \times \frac{h}{a\sqrt{2}} = \sqrt{2} \tan \alpha$.