



Graphs of exponential functions

About 85 minutes Calculator allowed 63 marks

Date _____

A Do Now

Quick questions to start.

- A1** [Needed today] [2 marks] **A2** [GCSE Higher] [2 marks]
Solve $3^x = \frac{1}{27}$. Work out the value of $8^{-\frac{2}{3}}$.

Answer

Answer

- A3** [Last lesson] [2 marks] **A4** [From chapter 3] [2 marks]
 $f(x) = 3x + 10$. Work out an expression for $f^{-1}(x)$. $f(x) = x^2 + 3$ for $-2 \leq x \leq 4$. Work out the range of f .

Answer

Answer

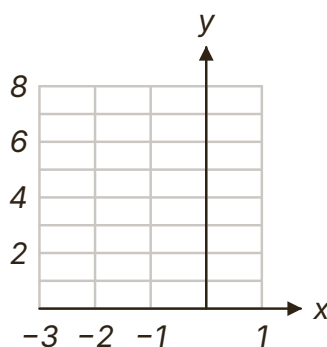
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

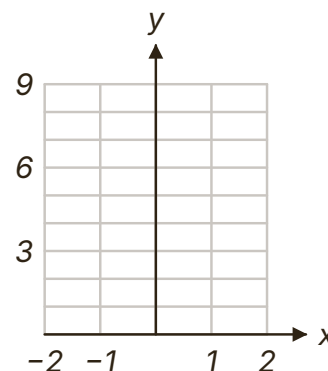
EXAMPLE 1

Work out the values of $y = 2^{-x}$ for $x = -3, -2, -1, 0, 1$. Draw the graph on the grid.



Hint 2^{-x} at $x = -3$ is 2^3 . At $x = 1$ it is $2^{-1} = \frac{1}{2}$. Join the points with a smooth curve, not straight lines.

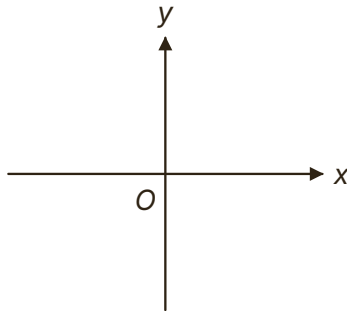
- B1** Work out the values of $y = 3^x$ [3 marks] for $x = -2, -1, 0, 1, 2$, as fractions where needed. Draw the graph on the grid.



Answer $y = \dots, \dots, \dots, \dots, \dots$

EXAMPLE 2

Sketch the graph of $y = 2^x + 3$ on the axes. Show where it crosses the y -axis and the line it gets closer to.

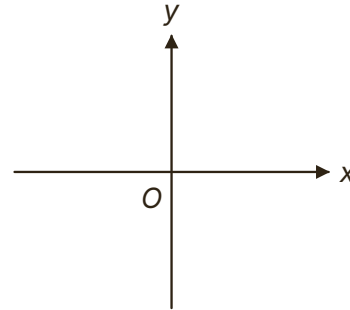


Hint $y = 2^x + 3$ is $y = 2^x$ moved up 3. At $x = 0$, $2^0 = 1$. As x gets very negative, 2^x gets closer to 0.

B2

Sketch the graph of $y = 3^{-x} - 2$ on the axes. Give the coordinates of the point where it crosses the y -axis and the equation of the line it gets closer to.

[3 marks]



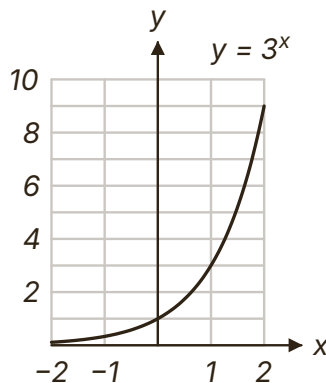
Answer (... , ...) $y = \dots\dots$

EXAMPLE 3

The graph of $y = 3^x$ is drawn on the grid.

(a) Use the graph to estimate the solution of $3^x = 5$.

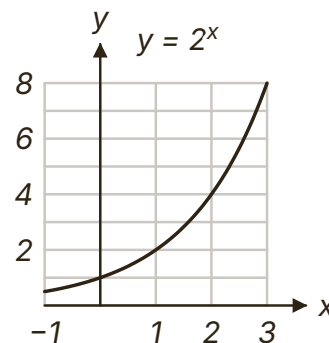
(b) By drawing a suitable straight line, estimate the solution of $3^x = 6 - x$.



Hint (a) Read across from $y = 5$ to the curve, then down. (b) Draw $y = 6 - x$; the solution is the x -coordinate where it meets the curve.

B3

The graph of $y = 2^x$ is drawn on the grid.



(a) Use the graph to estimate the solution of $2^x = 5$. [1 mark]

Answer $x = \dots\dots\dots$

(b) By drawing a suitable straight line on the grid, solve $2^x = 2x + 1$. Give both solutions. [3 marks]

Answer $x = \dots\dots$ and $x = \dots\dots$

Total for B3: 4 marks

EXAMPLE 4

The curve $y = ab^x$ passes through $(1, 10)$ and $(3, 250)$. Work out a and b .

Hint Put each point into $y = ab^x$. Divide one equation by the other: the a cancels.

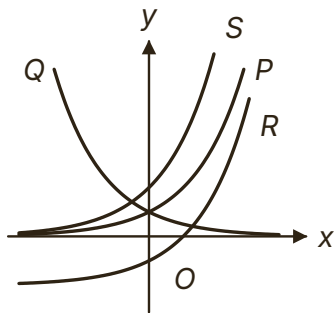
- B4** The curve $y = ab^x$ passes through $A(-1, 10)$ and $B(2, 0.08)$. Work out a and b . [4 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

c Practice

Level 2

- C1** The sketch shows the graphs of $y = 3^x$, $y = 3^{-x}$, $y = 3^x - 2$ and $y = 2 \times 3^x$. Match each equation to P , Q , R or S . Give the four letters in the order of the equations above. [2 marks]
- C2** The point $\left(-2, \frac{1}{25}\right)$ lies on the curve $y = a^x$, where $a > 0$. Work out a . [2 marks]



Answer $a = \dots\dots$

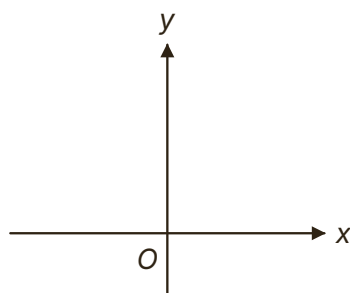
Answer

- C3** The number of cells in a sample after t hours is $N = 200 \times 3^t$. Work out the first whole number of hours after which there are more than 100 000 cells. [3 marks]
- C4** Work out the coordinates of the points where the curve $y = 2^x - 8$ crosses the x -axis and the y -axis. [2 marks]

Answer x -axis $(\dots, \dots)$ y -axis $(\dots, \dots)$

Answer hours

C5 Sketch $y = 3^x$ and $y = x^2$ on the axes. Hence state the number of solutions of $3^x = x^2$. [2 marks]



C6 The curve $y = 9 \times 3^{-x}$ passes through the point $(p, 81)$. Work out the value of p . [2 marks]

Answer $p = \dots\dots$

Answer $\dots\dots$ solution(s)

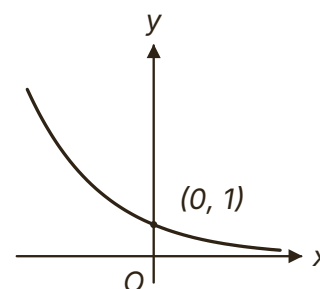
D Exam-style questions

AQA exam style

D1 Here is a sketch of a curve. Circle its equation. [1 mark]

$y = 2^x$ $y = 2^{-x}$ $y = -2^x$ $y = 2^x - 1$

Answer



D2 The curve $y = ab^{-x}$ passes through the points $(1, 1.5)$ and $(-1, 24)$, where a and b are positive constants.

(a) Work out the values of a and b . You must show your working. [4 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

(b) Work out the value of y when $x = -1.5$. [2 marks]

Answer $y = \dots\dots$

- (c) Hence show that the equation of the curve can be written as $y = 6 \times 0.25^x$. [2 marks]

Total for D2: 8 marks

D3 Josh says, "The curves $y = 3^x$ and $y = x^3$ only meet at $x = 3$."

Josh is wrong.

- (a) Work out $3^x - x^3$ when $x = 2$ and when $x = 2.5$. Use your answers to show that Josh is wrong. [2 marks]

- (b) Work out the x -coordinate of this other point, correct to 1 decimal place. You must show your working. [2 marks]

Answer $x = \dots\dots\dots$

Total for D3: 4 marks

D4 The number of fish in a lake t years after the start of 2020 is modelled by $N = 3000 \times 0.8^t$.

- (a) Work out the number of fish the model gives for the start of 2023. [2 marks]

Answer $\dots\dots$ fish

- (b) In which year does the number of fish first fall below 1000? [3 marks]

Answer $\dots\dots$

- (c) The number of fish in a second lake is modelled by $M = 500 \times 1.25^t$. Show that when $N = M$, $1.5625^t = 6$. [2 marks]

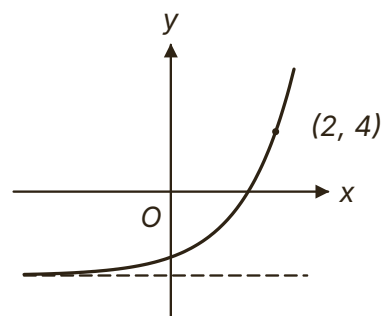
Total for D4: 7 marks

D5 The sketch shows the curve $y = 3^x + c$, where c is a constant. The curve passes through (2, 4). The dashed line $y = c$ is the line the curve gets closer to.

- (a) Work out the value of c . [1 mark]

Answer $c = \dots\dots$

- (b) Work out the coordinates of the points where the curve crosses the axes. Give the x -coordinate correct to 2 decimal places. [3 marks]



Not drawn accurately

Answer $(\dots, \dots)$ and $(\dots, \dots)$

Total for D5: 4 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Work out the coordinates of the points where the curve $y = 4^x - 10 \times 2^x + 16$ crosses the x -axis. [4 marks]

Answer $(\dots, \dots)$ and $(\dots, \dots)$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** $4^x = (2^x)^2$. Let $u = 2^x$.
Hint 2 The equation becomes $u^2 - 10u + 16 = 0$.
Hint 3 $(u - 2)(u - 8) = 0$, so $2^x = 2$ or $2^x = 8$, giving $x = 1$ or $x = 3$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can work out values of a^x and a^{-x} and draw the graph. ☐ ☐ ☐

I can sketch graphs such as $y = 3^x + k$ and $y = 3^{-x}$, showing where they cross the y -axis. ☐ ☐ ☐

I can find a and b in $y = ab^x$ from two points. ☐ ☐ ☐

I can solve equations with a^x from a graph and use exponential models. ☐ ☐ ☐