

Graphs of functions and linear functions

About 75 minutes No calculator 62 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-graphs-of-functions-and-linear-functions/

Date _____

A Do Now

The first two questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Rearrange $3x - 2y = 8$ to make y the subject.

A2 Needed today [2 marks]

The point $(k, 19)$ lies on the line $y = 4 - 3x$. Work out k .

Answer $y = \dots\dots$

Answer $k = \dots\dots$

- A3** Last lesson [2 marks]

$f(x) = 3x + 1$ and $g(x) = x^2 - 4$.
Work out $gf(-2)$.

A4 From chapter 2 [2 marks]

Make x the subject of $3(x - a) = bx + 2$.

Answer $gf(-2) = \dots\dots$

Answer $x = \dots\dots$

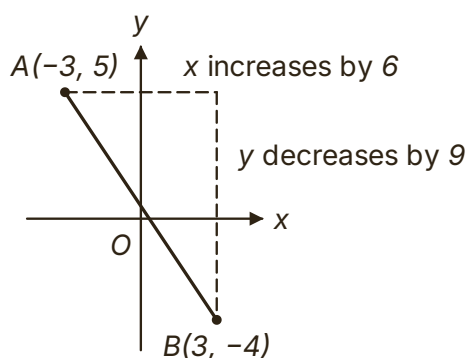
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Work out the gradient of the line through $A(-3, 5)$ and $B(3, -4)$. Does the line go uphill or downhill from left to right?



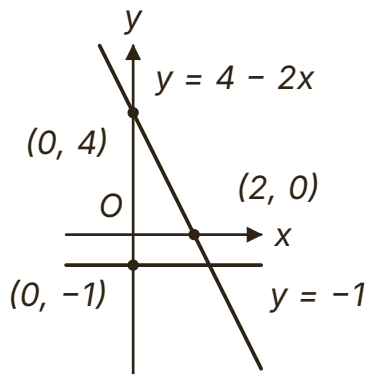
Hint Gradient = $\frac{\text{change in } y}{\text{change in } x}$. Take both changes from A to B .

- B1** [3 marks]
- Work out the gradient of the line through $P(-2, 9)$ and $Q(4, -6)$, and the gradient of the line through $R(-5, 1)$ and $S(3, 7)$. Which of the two lines goes downhill from left to right?

Answer $PQ : \dots\dots$ $RS : \dots\dots$ downhill: $\dots\dots$

EXAMPLE 2

Sketch the lines $y = 4 - 2x$ and $y = -1$ on the same axes. Mark the coordinates of each point where a line meets an axis.



Hint For $y = mx + c$, put $x = 0$ and then $y = 0$. The line $y = -1$ is parallel to the x -axis.

- B2** Sketch the lines $y = 3x - 6$ and $x = -2$ on the same axes. Mark the coordinates of each point where a line meets an axis. [3 marks]

EXAMPLE 3

The line L has equation $5x - 3y + 15 = 0$. Work out where L meets each axis, and the gradient of L .

Hint Put $x = 0$, then $y = 0$. For the gradient, make y the subject.

- B3** The line M has equation $2x + 5y - 20 = 0$. Work out where M meets each axis, and the gradient of M . [3 marks]

Answer x - $(\dots, \dots)$ y - $(\dots, \dots)$ gradient $\dots\dots$
axis axis

EXAMPLE 4

A line has equation $\frac{x}{4} - \frac{y}{5} - 1 = 0$. Write it in the form $px + qy + r = 0$, where p , q and r are integers, and write down where it meets each axis.

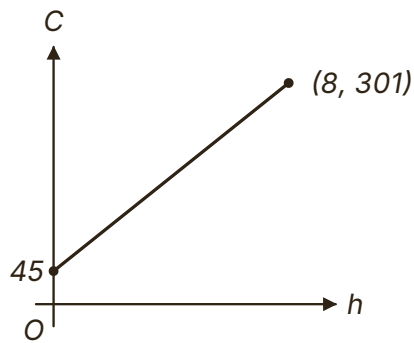
Hint Multiply every term by 20. The form $\frac{x}{a} + \frac{y}{b} = 1$ shows the crossings: $(a, 0)$ and $(0, b)$.

- B4** Write $6x - 5y = 30$ in the form $\frac{x}{a} + \frac{y}{b} = 1$. Hence write down where the line meets each axis. [3 marks]

Answer $a = \dots\dots$ $b = \dots\dots$ $(\dots, \dots)$ and $(\dots, \dots)$

EXAMPLE 5

A plumber charges $\pounds C$ for a job that lasts h hours, where $C = 45 + 32h$. No job lasts more than 8 hours. Work out the charge for a 3-hour job and the cost per hour of that job. State the fixed charge and the charge per hour, and sketch C against h .



Hint The constant term is the charge for 0 hours. The coefficient of h is the gradient: the extra charge for each hour.

B5 A phone plan costs $\pounds C$ in a month when g gigabytes of data are used, where $C = 12 + 1.5g$. Work out the cost for 10 gigabytes and for 40 gigabytes, and the cost per gigabyte in each case. State the fixed monthly charge. [4 marks]

Answer $\pounds$, $\pounds$; $\pounds$, $\pounds$; $\pounds$
per fixed
GB

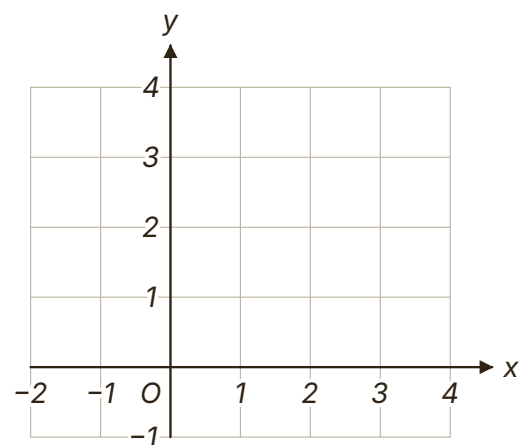
c Practice

Level 2

- C1** For each line, say whether its gradient is positive, negative, zero or infinite. [2 marks]
 (i) $y = 5 - x$
 (ii) $x = -4$ (iii) $y = 2$ (iv) $3y = x$

Answer (i) (ii) (iii) (iv)

- C2** $f(x) = \frac{1}{2}x + 1$ with domain $-2 \leq x \leq 4$. Draw the graph of $y = f(x)$ on the grid. Then write down the range of $f(x)$. [3 marks]



Answer $\leq f(x) \leq$

- C3** The line $2x + 7y = k$ passes through the point $(-3, 2)$. Work out the value of k . Then work out where the line meets the x -axis. [3 marks]

Answer $k = \dots\dots$ $(\dots, \dots)$

- C4** Show that the points $A(-2, -1)$, $B(1, 5)$ and $C(4, 11)$ lie on one straight line. [2 marks]

- C5** Write the equation $y = \frac{2}{3}x - 5$ in the form $px + qy + r = 0$, where p , q and r are integers. [2 marks]

Answer $\dots\dots = 0$

D Exam-style questions

AQA exam style

- D1** Which line has gradient $-\frac{3}{2}$? Circle your answer. [1 mark]
- $2x + 3y = 6$ $3x + 2y = 6$ $3x - 2y = 6$ $2x - 3y = 6$

- D2** The points $A(p, 3p)$ and $B(4, 2)$ lie on a straight line with gradient 4.

- (a) Work out the value of p . [3 marks]

Answer $p = \dots\dots$

- (b) The line meets the y -axis at C . Work out the coordinates of C . [2 marks]

Answer $(\dots\dots, \dots\dots)$

Total for D2: 5 marks

D3

The line L has equation $2x - 5y + 15 = 0$. Sam says, "The gradient of L is 2, and L crosses the y -axis at $(0, 15)$."

Sam is wrong.

- (a) Work out the gradient of L .

[2 marks]

Answer

- (b) Write down where L crosses the y -axis, and explain Sam's mistake.

[2 marks]

Answer (...,...)

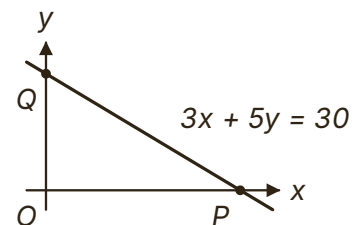
Total for D3: 4 marks

D4 The line $3x + 5y = 30$ meets the x -axis at P and the y -axis at Q . O is the origin.

- (a) Work out the coordinates of P and of Q .

[2 marks]

Answer $P(...,...)$ $Q(...,...)$



Not drawn accurately

- (b) Work out the area of triangle OPQ .

[1 mark]

Answer square units

- (c) A straight line through O meets PQ at M and cuts triangle OPQ into two triangles of equal area. Work out the gradient of OM .

[3 marks]

Answer

Total for D4: 6 marks

D5 Water drains from a tank. The volume, V litres, left in the tank t minutes after the tap is opened is $V = 480 - 15t$. The model holds until the tank is empty.

- (a) Write down the volume when the tap is opened, and the rate at which the volume falls.

[2 marks]

Answer litres; litres per minute

(b) Work out the value of t when the volume is 180 litres.

[2 marks]

Answer $t = \dots\dots$

(c) Sketch the graph of V against t for the values of t where the model holds.
Mark where your graph meets the axes.

[2 marks]

Total for D5: 6 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A is the point $(1, 1)$ and B is the point $(3, 7)$. C is the point $(k, k^2 - 2)$ and is not the same point as B . A , B and C lie on one straight line. Work out the coordinates of C . [4 marks]

Answer $(\dots\dots, \dots\dots)$

Extension hints

Cover these with paper. Uncover one at a time.

E1 **Hint 1** Work out the gradient of AB . The gradient of AC must be the same.

Hint 2 Gradient $AB = 3$, so $\frac{k^2 - 2 - 1}{k - 1} = 3$.

Hint 3 $k^2 - 3 = 3k - 3$, so $k^2 - 3k = 0$ and $k = 0$ or $k = 3$.
 $k = 3$ gives $(3, 7)$, which is B , so $k = 0$ and C is $(0, -2)$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can work out the gradient of a line from two points and say what its sign means.

☐ ☐ ☐

I can sketch a straight line from any form of its equation, marking where it meets the axes.

☐ ☐ ☐

I can use the form $\frac{x}{a} + \frac{y}{b} = 1$ and rewrite equations with integer coefficients.

☐ ☐ ☐

I can use a linear model, explaining its gradient and fixed term and the domain where it holds.

☐ ☐ ☐