



Graphs of quadratic functions

About 85 minutes No calculator 62 marks

Date _____

A Do Now

The first two questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks] **A2** Needed today [2 marks]
- Write $x^2 - 8x + 13$ in the form $(x + a)^2 + b$. Solve $x^2 + 2x - 15 = 0$.

Answer $x = \dots\dots$ or $x = \dots\dots$

Answer $\dots\dots$

- A3** Last lesson [2 marks] **A4** From chapter 1 [2 marks]
- Work out the equation of the line through $(1, 6)$ and $(5, -2)$. Expand and simplify $(2 + \sqrt{3})^2$.
Give it in the form $y = mx + c$.

Answer $\dots\dots$

Answer $y = \dots\dots$

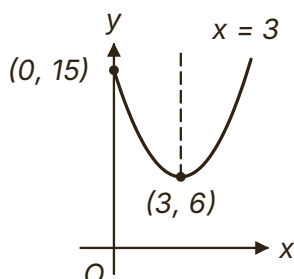
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

For the graph of $y = x^2 - 6x + 15$, work out the vertex, the equation of the line of symmetry and where the graph meets the y -axis. Sketch the graph.



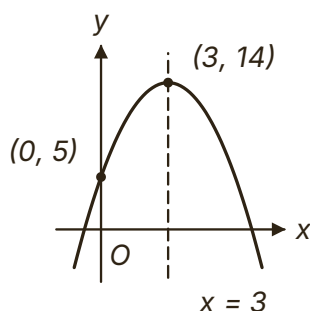
Hint Complete the square. $(x - p)^2$ is never negative, so the least value is when the bracket is 0.

- B1** [3 marks] For the graph of $y = x^2 + 12x + 43$, work out the vertex, the equation of the line of symmetry and where the graph meets the y -axis.

Answer vertex $(\dots, \dots)$ $x = \dots$ $(\dots, \dots)$

EXAMPLE 2

For the graph of $y = 5 + 6x - x^2$, write y in the form $k - (x - p)^2$. Hence write down the vertex and the line of symmetry, and sketch the graph.



Hint Take out -1 : $-(x^2 - 6x) + 5$. Complete the square inside the bracket.

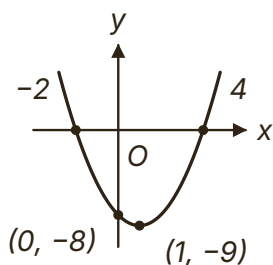
B2 For the graph of [3 marks]

$y = 8x - x^2 - 1$, write y in the form $k - (x - p)^2$. Hence write down the vertex and say whether it is a maximum or a minimum point.

Answer $y = \dots\dots (\dots, \dots)$, $\dots\dots$

EXAMPLE 3

Sketch the graph of $y = x^2 - 2x - 8$. Mark the coordinates of the points where it meets the axes, and of its vertex.



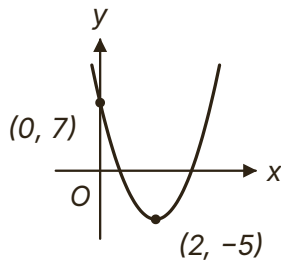
Hint Factorise to find where $y = 0$. The vertex is halfway between the two roots.

B3 Sketch the graph of [4 marks]

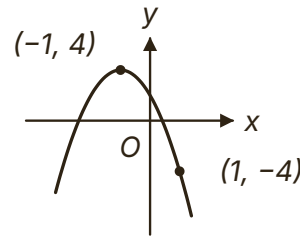
$y = x^2 + 2x - 24$. Mark the coordinates of the points where it meets the axes, and of its vertex.

EXAMPLE 4

A quadratic curve has its vertex at $(2, -5)$ and passes through $(0, 7)$. Work out its equation in the form $y = ax^2 + bx + c$.



Hint A curve with vertex (p, q) has equation $y = a(x - p)^2 + q$. Use the other point to find a .

B4 The curve in the diagram has its vertex at $(-1, 4)$ and passes through $(1, -4)$. Work out its equation in the form $y = ax^2 + bx + c$. [3 marks]

Answer $y = \dots\dots$

EXAMPLE 5

A farmer has 40 m of fencing to make three sides of a rectangular pen. A wall is the fourth side. The two sides at right angles to the wall are x m long. Show that the area is $A = 40x - 2x^2$, and work out the greatest possible area.

Hint The side parallel to the wall is $40 - 2x$. Complete the square on $40x - 2x^2$.

B5 A shop's daily profit is $\mathcal{L}P$ when it charges $\mathcal{L}x$ for an item, where $P = 18x - x^2 - 45$. Write P in the form $k - (x - p)^2$. Hence work out the greatest daily profit and the price that gives it. Work out the two prices at which the profit is $\mathcal{L}0$. [4 marks]

Answer $\mathcal{L}\dots\dots$ at $\mathcal{L}\dots\dots$; $\mathcal{L}\dots\dots$ and $\mathcal{L}\dots\dots$

c Practice

Level 2

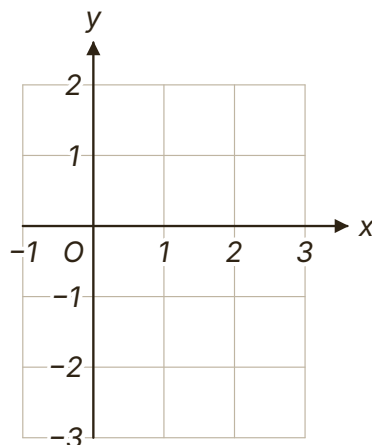
C1 $f(x) = x^2 + 2x - 4$ with domain $-4 \leq x \leq 1$. Work out the range of $f(x)$. [3 marks]

C2 Write $x^2 - 2x - 6$ in the form $(x + a)^2 + b$. Hence work out the exact values of x where the graph of $y = x^2 - 2x - 6$ meets the x -axis. [3 marks]

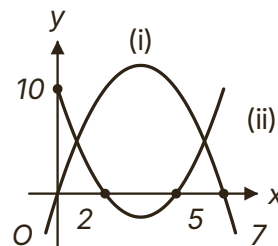
Answer $\dots\dots \leq f(x) \leq \dots\dots$

Answer $\dots\dots$ $x = \dots\dots$

- C3** Draw the graph of $y = x^2 - 2x - 2$ for $-1 \leq x \leq 3$ on the grid. Then write down the range of y for this domain.



- [3 marks] **C4** The equations of curves (i) and (ii) are two of $y = 7x - x^2$, $y = x^2 - 7x$, $y = x^2 - 7x + 10$ and $y = 10 + 7x - x^2$. Choose the equation of each curve.



Answer (i) (ii)

Answer $\leq y \leq$

D Exam-style questions

AQA exam style

- D1** Zak says, "The least value of $x^2 + 6x + 16$ is 16, because when $x = 0$ the value is 16."

Zak is wrong.

- (a) Write $x^2 + 6x + 16$ in the form $(x + a)^2 + b$. [2 marks]

Answer

- (b) Hence write down the least value, and the value of x that gives it. Explain Zak's mistake. [2 marks]

Answer least value when $x =$

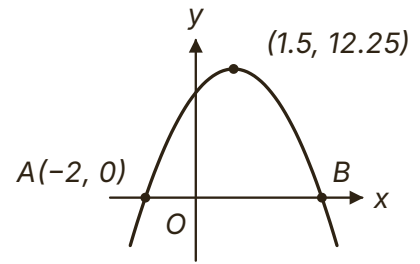
Total for D1: 4 marks

D2 Here is a sketch of $y = f(x)$, where $f(x)$ is a quadratic function. The graph meets the x -axis at $A(-2, 0)$ and at B , and has a maximum point at $(1.5, 12.25)$.

- (a) Work out the coordinates of B . [1 mark]

Answer $(\dots, \dots)$

- (b) The equation $f(x) = k$ has exactly one solution. Write down the value of k . [1 mark]



Not drawn accurately

Answer $k = \dots\dots$

- (c) The graph meets the y -axis at $(0, 10)$. Work out $f(x)$ in the form $ax^2 + bx + c$. [3 marks]

Answer $f(x) = \dots\dots$

Total for D2: 5 marks

D3 The least value of $x^2 + 8x + c$ is -9 .

- (a) Work out the value of c . [3 marks]

Answer $c = \dots\dots$

- (b) Hence work out where the graph of $y = x^2 + 8x + c$ meets the x -axis. [2 marks]

Answer $(\dots, \dots)$ and $(\dots, \dots)$

- (c) Write down the values of k for which $x^2 + 8x + c = k$ has two solutions. [1 mark]

Answer $\dots\dots$

Total for D3: 6 marks

D4 A stone is thrown upwards from the top of a cliff. Its height above the sea, h metres, t seconds after it is thrown is $h = 35 + 30t - 5t^2$.

- (a) Show that $h = 80 - 5(t - 3)^2$. [2 marks]

(b) Write down the greatest height of the stone and the time when it reaches it. [2 marks]

Answer m at $t = \dots\dots$

(c) Work out the time when the stone reaches the sea. [2 marks]

Answer $t = \dots\dots$

Total for D4: 6 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The curve $y = x^2 + bx + 8$ meets the y -axis at $(0, 8)$. Its vertex lies on the line $y = 2x$. Work out the two possible values of b , and the vertex of each curve. [5 marks]

Answer $b = \dots\dots$, $(\dots, \dots)$; $b = \dots\dots$, $(\dots, \dots)$

Extension hints

Cover these with paper. Uncover one at a time.

E1 **Hint 1** Complete the square to write the vertex in terms of b .

Hint 2 $y = \left(x + \frac{b}{2}\right)^2 + 8 - \frac{b^2}{4}$, so the vertex is $\left(-\frac{b}{2}, 8 - \frac{b^2}{4}\right)$. Put it into $y = 2x$.

Hint 3 $8 - \frac{b^2}{4} = -b$, so $b^2 - 4b - 32 = 0$ and $(b - 8)(b + 4) = 0$. $b = 8$ gives the vertex $(-4, -8)$; $b = -4$ gives the vertex $(2, 4)$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the vertex and line of symmetry of a parabola by completing the square.

☐ ☐ ☐

I can sketch a quadratic graph, marking its intercepts and vertex.

☐ ☐ ☐

I can find the equation of a parabola from its vertex, or its roots, and one other point.

☐ ☐ ☐

I can use a quadratic model to find a greatest or least value.

☐ ☐ ☐