

Increasing and decreasing functions, and the second derivative

About 75 minutes No calculator 71 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-increasing-and-decreasing-functions-and-the-second-derivative/

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Solve $x^2 - 3x - 18 < 0$.

A2 Needed today [2 marks]

Write $3x^2 - 12x + 20$ in the form $a(x + b)^2 + c$.

Answer $\dots\dots < x < \dots\dots$

Answer

- A3** Last lesson [2 marks]

Work out the gradient of the normal to the curve $y = x^3 - 2x$ at the point where $x = 2$.

A4 From chapter 4 [2 marks]

Simplify $\frac{(2x^3)^2}{4x}$.

Answer

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Work out the values of x for which $y = x^2 - 6x + 1$ is an increasing function.

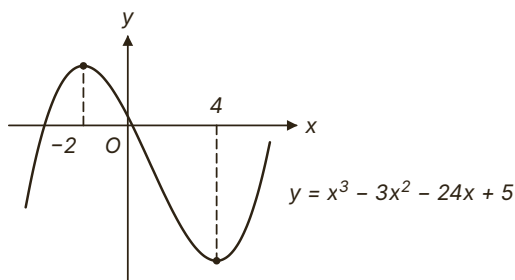
Hint A function is increasing where $\frac{dy}{dx} > 0$.

- B1** [3 marks]
- Work out the values of x for which $y = 7 + 10x - x^2$ is a decreasing function.

Answer $x \dots\dots$

EXAMPLE 2

Work out the values of x for which $y = x^3 - 3x^2 - 24x + 5$ is a decreasing function.



Not drawn accurately

Hint Solve $\frac{dy}{dx} < 0$. Find the critical values first, then decide which side of them the gradient is negative.

B2 Work out the values of x for which $y = 2x^3 + 3x^2 - 36x$ is an increasing function. [4 marks]

Answer

EXAMPLE 3

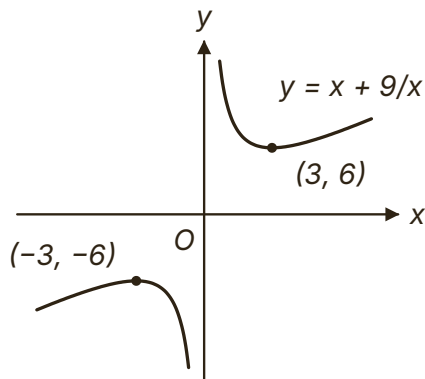
Prove that $y = x^3 + 3x^2 + 5x - 2$ is an increasing function for all values of x .

Hint Show that $\frac{dy}{dx} > 0$ for every x . Complete the square on $\frac{dy}{dx}$.

B3 Prove that $y = 1 - 5x + 3x^2 - x^3$ is a decreasing function for all values of x . [4 marks]

EXAMPLE 4

Work out the values of x for which $y = x + \frac{9}{x}$ is an increasing function.



Not drawn accurately

Hint Write $y = x + 9x^{-1}$. The function is not defined at $x = 0$.

B4 Work out the values of x for [4 marks]

which $y = 4x + \frac{25}{x}$ is a decreasing function.

Answer

EXAMPLE 5

$y = (2x - 1)(x^2 + 3)$. Work out $\frac{d^2y}{dx^2}$. Hence work out the rate of change of the gradient of the curve at the point where $x = 2$.

Hint Multiply out first. $\frac{d^2y}{dx^2}$ is the derivative of $\frac{dy}{dx}$: it is the rate of change of the gradient.

B5 $y = (x^2 - 4)(3x + 1)$. Work out [4 marks]

$\frac{d^2y}{dx^2}$. Hence work out the rate of change of the gradient of the curve at the point where $x = -1$.

Answer $\frac{d^2y}{dx^2} = \dots\dots\dots$ and $\dots\dots\dots$

c Practice

Level 2

C1 Work out the values of x for [2 marks]
which $y = 2x^2 + 9x - 5$ is a decreasing function.

Answer $x \dots\dots\dots$

C2 Prove that $y = \frac{1}{3}x^3 - 3x^2 + 11x$ [3 marks]
is an increasing function for all values of x .

C3 $y = 2x^3 - \frac{8}{x}$. Work out the value of $\frac{d^2y}{dx^2}$ when $x = 1$. [3 marks]

Answer

C4 Work out the values of x for which the gradient of the curve $y = x^3 - 6x^2 + 4$ is increasing. [3 marks]

Answer x

C5 $y = x^4 - 6x^2 + 2$. Work out the values of x for which $\frac{d^2y}{dx^2} = 0$. [3 marks]

Answer $x = \dots\dots$ and $x = \dots\dots$

D Exam-style questions

AQA exam style

D1 Kai is working out the values of x for which $y = x^3 - 27x$ is a decreasing function. He writes: $3x^2 - 27 < 0$, so $x^2 < 9$, so $x < 3$. [3 marks]

Explain Kai's mistake. Work out the correct values of x .

Answer $\dots\dots < x < \dots\dots$

D2 $f(x) = 2x^3 - 6x^2 + 9x + 1$. Use differentiation to show that $f(x)$ is an increasing function for all values of x . [4 marks]

D3 Work out the values of x for which $f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 6x$ is a decreasing function. Give your answer as an inequality. [5 marks]

Answer $\dots\dots < x < \dots\dots$

D4 A ball is thrown upwards. Its height, h metres, above the ground t seconds after it is thrown is $h = 1.5 + 14t - 5t^2$.

(a) Work out $\frac{dh}{dt}$.

[1 mark]

Answer $\frac{dh}{dt} = \dots\dots\dots$

(b) Work out the values of t for which the height of the ball is increasing.

[2 marks]

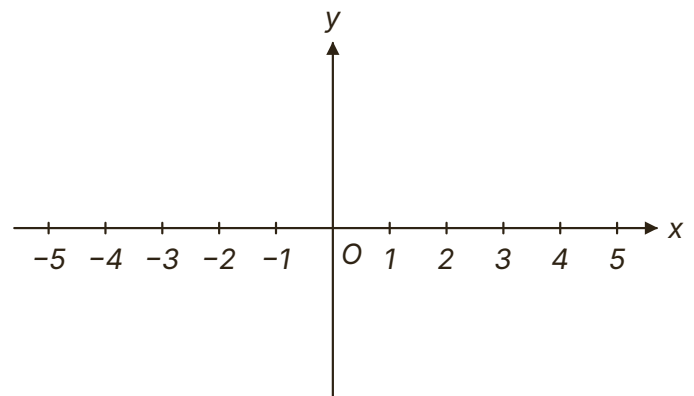
Answer $\dots\dots\dots$

(c) Work out $\frac{d^2h}{dt^2}$. Say what it tells you about the ball.

[2 marks]

Total for D4: 5 marks

D5 $y = f(x)$ is the graph of a cubic function. $f(x) > 0$ for $x < 4$ and $f(x) \leq 0$ for $x \geq 4$. [4 marks]
The function is decreasing for $x < -3$, increasing for $-3 < x < 1$ and decreasing for $x > 1$. Draw a possible sketch of $y = f(x)$ for values of x from -5 to 5 on the axes.



D6 $y = x^3 - 6x^2 + kx + 2$, where k is an integer. $\frac{dy}{dx} > 0$ for all values of x . Work out the least possible value of k . [4 marks]

Answer $k = \dots\dots\dots$

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Work out the values of x for which $y = x^2 + \frac{16}{x}$ is a decreasing function. [5 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1 Hint 1** Work out $\frac{dy}{dx}$ and solve $\frac{dy}{dx} < 0$. Be careful when you multiply an inequality by something that might be negative.
- Hint 2** $\frac{dy}{dx} = 2x - \frac{16}{x^2}$. Multiply by x^2 , which is positive for every $x \neq 0$.
- Hint 3** $2x^3 - 16 < 0$, so $x^3 < 8$ and $x < 2$. The function is not defined at $x = 0$, so it is decreasing for $x < 0$ and for $0 < x < 2$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can work out where a function is increasing or decreasing. ☐ ☐ ☐
- I can prove that a function is increasing, or decreasing, for all values of x . ☐ ☐ ☐
- I can work out the second derivative and the rate of change of the gradient. ☐ ☐ ☐
- I can say what $\frac{dh}{dt}$ and $\frac{d^2h}{dt^2}$ mean in a context. ☐ ☐ ☐