



Indices

About 80 minutes No calculator 76 marks

Date _____

A Do Now

Quick questions to start.

A1 GCSE Higher [2 marks]

Work out the exact value of $\left(\frac{16}{9}\right)^{-\frac{3}{2}}$.

A2 Needed today [2 marks]

Solve $x^2 - 10x + 21 = 0$.

Answer $x = \dots\dots$ or $x = \dots\dots$

Answer

A3 Last lesson [2 marks]

Solve $2x^2 - 7x - 4 \leq 0$.

A4 From chapter 3 [2 marks]

The graph of $y = a \times 3^x$ passes through the point (2, 36). Work out the value of a .

Answer

Answer $a = \dots\dots$

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Write $\frac{\sqrt[3]{x^2} \times x^4}{(x^2)^{-\frac{1}{3}}}$ as a single power of x .

Hint Write every part as a power of x first: a cube root is the power $\frac{1}{3}$. Then add the indices on top and subtract the index underneath.

B1 [3 marks]

Write $\frac{(x^3)^{\frac{1}{4}} \times \sqrt[4]{x^5}}{x^{-\frac{1}{2}}}$ as a single power of x .

Answer

EXAMPLE 2

Solve $x^{-\frac{5}{2}} = \frac{1}{243}$

Hint Take the reciprocal of both sides to make the index positive. Then undo the power $\frac{5}{2}$ one step at a time.

B2 [3 marks]

Solve $x^{-\frac{3}{4}} = \frac{8}{27}$

Answer $x = \dots\dots$

EXAMPLE 3

Expand and simplify $\left(x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}\right)^2$

Hint Write it as two brackets multiplied. When you multiply powers of x , add the indices:
 $x^{\frac{1}{2}} \times x^{-\frac{1}{2}} = x^0 = 1$.

B3 Expand and simplify [3 marks]

$$\left(2x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)\left(x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}\right)$$

Answer

EXAMPLE 4

Solve $x^4 - 11x^2 + 18 = 0$. Give your answers in exact form.

Hint There are no odd powers of x . Let $y = x^2$, so $x^4 = y^2$.

B4 Solve $2x^4 + 5x^2 - 12 = 0$. Give your answers in exact form. [4 marks]

Answer $x = \dots\dots\dots$

EXAMPLE 5

Solve $9^x - 6 \times 3^x - 27 = 0$

Hint $9^x = (3^2)^x = (3^x)^2$, so let $y = 3^x$. Remember that 3^x can never be negative.

B5 Solve $4^{x+1} - 33 \times 2^x + 8 = 0$ [5 marks]

Answer $x = \dots\dots$ or $x = \dots\dots$

c Practice

Level 2

C1 Write both sides as powers of the same number. Then solve each equation.

(a) $8^x = 4^{x+3}$ [2 marks]

Answer $x = \dots\dots$

(b) $25^x = \frac{1}{125}$ [2 marks]

Answer $x = \dots\dots$

Total for C1: 4 marks

C2 Solve $2x - 9\sqrt{x} + 4 = 0$ [4 marks]

Answer $x = \dots\dots$ or $x = \dots\dots$

C3 Solve $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 4 = 0$

[4 marks]

C4 Solve the simultaneous equations

[4 marks]

$$2^x \times 4^y = 32 \quad \text{and} \quad 3^x \div 9^y = \frac{1}{27}$$

Answer $x = \dots\dots$ or $x = \dots\dots$

C5 Solve $x^6 + 7x^3 - 8 = 0$

[4 marks]

C6 Write $\frac{3x^2 - 5x^{\frac{1}{2}}}{x^{\frac{3}{2}}}$ in the form $ax^p + bx^q$

[3 marks]

Answer $x = \dots\dots$, $y = \dots\dots$

Answer $x = \dots\dots$ or $x = \dots\dots$

Answer $\dots\dots\dots$

D Exam-style questions

AQA exam style

D1 $\frac{y^{\frac{5}{2}} \times y^{\frac{1}{2}}}{y^{-1}} = y^m$

[1 mark]

Circle the value of m .

2 3 4 $\frac{5}{4}$

D2 $a^n \times (a^3)^n = \frac{a^{20}}{a^n}$

[2 marks]

Work out the value of n .

Answer $n = \dots\dots$

D3 Ali says, " 9^x is the same as 3×3^x ."

Ali is wrong.

(a) Show that Ali is wrong.

[1 mark]

(b) Solve $9^x + 3^x = 90$. You must show your working.

[4 marks]

Answer $x = \dots\dots$

Total for D3: 5 marks

D4

$$\frac{x^2}{2} + \frac{8}{x^2} = 5$$

(a) Show that the equation can be written as $x^4 - 10x^2 + 16 = 0$

[2 marks]

(b) Hence solve $\frac{x^2}{2} + \frac{8}{x^2} = 5$. Give your answers in exact form.

[3 marks]

Answer $x = \dots\dots\dots$

Total for D4: 5 marks

D5 Solve $2^{x^2} \times 4^{x-3} = 8^x$

[5 marks]

You must show your working.

Answer $x = \dots\dots$ or $x = \dots\dots$

D6 Solve $2x^{\frac{1}{2}} + 15x^{-\frac{1}{2}} = 11$

[4 marks]

Answer $x = \dots\dots$ or $x = \dots\dots$

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Find every value of x for which $(x^2 - 7x + 11)^{x-2} = 1$ [5 marks]

Answer $x = \dots\dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** A power a^b equals 1 in three ways: $a = 1$; or $b = 0$ (with $a \neq 0$); or $a = -1$ and b is even.
- Hint 2** Base 1: $x^2 - 7x + 10 = 0$. Index 0: $x = 2$. Base -1 : $x^2 - 7x + 12 = 0$, then check the index is even.
- Hint 3** Base 1 gives $x = 2$ or 5 . Index 0 gives $x = 2$ again. Base -1 gives $x = 3$ (index 1, odd: rejected) or $x = 4$ (index 2, even). So $x = 2, 4$ or 5 .

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can use the index laws with fractional and negative powers. ☐ ☐ ☐
- I can solve an equation with a fractional index. ☐ ☐ ☐
- I can solve a quadratic in disguise. ☐ ☐ ☐
- I can solve an equation with the unknown in the index. ☐ ☐ ☐