



Inverse functions

About 80 minutes No calculator 70 marks

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks]

Make x the subject of

$$y = \frac{3x - 4}{5}.$$

A2 Needed today [2 marks]

Make x the subject of

$$y = 2x^2 + 7, \text{ where } x > 0.$$

Answer _____

Answer _____

- A3** Last lesson [2 marks]

Work out the coordinates of the turning point of

$$y = x^2 + 8x + 11.$$

A4 From chapter 3 [2 marks]

$f(x) = x^2 - 1$ and $g(x) = 2x + 3$.
 Work out $fg(-2)$.

Answer _____

Answer (.....,)

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$f(x) = 4x - 7$. Work out an expression for $f^{-1}(x)$.

Hint Write $y = 4x - 7$, swap x and y , then make y the subject.

- B1** $f(x) = \frac{5x + 2}{3}$. Work out an expression for $f^{-1}(x)$. [2 marks]

Answer $f^{-1}(x) = \dots\dots\dots$

EXAMPLE 2

$f(x) = x^2 + 3$ for $x \geq 0$. Work out an expression for $f^{-1}(x)$.

Hint Swap x and y . Only the positive square root is used, because $x \geq 0$ in f .

- B2** $f(x) = 2x^2 - 5$ for $x \geq 0$. Work out an expression for $f^{-1}(x)$. [3 marks]

Answer $f^{-1}(x) = \dots\dots\dots$

EXAMPLE 3

$f(x) = (x - 3)^2 + 1$ for $x \geq 3$. Work out $f^{-1}(x)$ and state the domain of f^{-1} .

Hint The domain of f^{-1} is the range of f . The least value of $(x - 3)^2 + 1$ is 1.

B3 $f(x) = (x + 2)^2 - 6$ for $x \geq -2$. [4 marks]

Work out $f^{-1}(x)$ and state the domain of f^{-1} .

Answer $f^{-1}(x) = \dots\dots\dots$ domain $\dots\dots\dots$

EXAMPLE 4

$f(x) = \frac{6}{x-1}$ for $x > 1$. Work out $f^{-1}(x)$ and state the domain of f^{-1} .

Hint For $x > 1$ the denominator is positive, so every value of $f(x)$ is positive.

B4 $f(x) = \frac{12}{x+3}$ for $x > 0$. Work [4 marks]

out $f^{-1}(x)$ and state the domain of f^{-1} .

Answer $f^{-1}(x) = \dots\dots\dots$ domain $\dots\dots\dots$

EXAMPLE 5

$f(x) = 3x - 8$. Solve $f(x) = f^{-1}(x)$.

Hint Find $f^{-1}(x)$ first. Then put the two expressions equal.

B5 $f(x) = 5x + 12$. Solve [3 marks]

$f(x) = f^{-1}(x)$.

Answer $x = \dots\dots\dots$

c Practice

Level 2

C1 $f(x) = 7 - 2x$. Work out $f^{-1}(3)$ [2 marks]
by first finding an expression for $f^{-1}(x)$.

Answer $f^{-1}(3) = \dots\dots\dots$

C2 $f(x) = \sqrt{x+4}$ for $x \geq -4$. Work [3 marks]
out $f^{-1}(x)$ and state the domain of f^{-1} .

Answer $f^{-1}(x) = \dots\dots\dots$ domain $\dots\dots\dots$

- C3** $f(x) = 2x^3 + 1$. Work out $f^{-1}(x)$. [3 marks]
Hence work out $f^{-1}(55)$.
- C4** $f(x) = (x - 4)^2$ for all values of x . Explain why f has no inverse function. Then give a domain of the form $x \geq k$ on which f does have an inverse. [2 marks]

Answer $f^{-1}(x) = \dots\dots\dots$ $f^{-1}(55) = \dots\dots\dots$

Answer domain $x \geq \dots\dots\dots$

- C5** $f(x) = 3x + a$, where a is a constant. $f^{-1}(14) = 4$. Work out the value of a . [2 marks]
- C6** $g(x) = \frac{x+1}{x-2}$ for $x \neq 2$. Work out an expression for $g^{-1}(x)$. [3 marks]

Answer $a = \dots\dots\dots$

Answer $g^{-1}(x) = \dots\dots\dots$

D Exam-style questions

AQA exam style

- D1** $f(x) = \frac{x-6}{2}$. Circle the expression for $f^{-1}(x)$. [1 mark]
- $2x - 6$ $2x + 6$ $\frac{x+6}{2}$ $\frac{2}{x-6}$

Answer

- D2** $f(x) = x^2 - 9$ for $x \geq 0$. Priya says that $f^{-1}(x) = \sqrt{x} + 3$, because $\sqrt{9} = 3$.

Priya is wrong.

- (a) Use $f(4)$ to show that Priya is wrong. [1 mark]

- (b) Work out the correct expression for $f^{-1}(x)$ and state its domain. [3 marks]

Answer $f^{-1}(x) = \dots\dots\dots$ domain $\dots\dots\dots$

Total for D2: 4 marks

D3 $f(x) = x^2 + 5$ for $x \geq 0$

$g(x) = 3x - 1$ for all x

(a) Work out $f^{-1}(x)$ and state its domain.

[3 marks]

Answer $f^{-1}(x) = \dots\dots\dots$ domain $\dots\dots\dots$

(b) Show that $g^{-1}f(x)$ can be written as $\frac{x^2 + a}{3}$, where a is an integer.

[2 marks]

Answer $a = \dots\dots\dots$

(c) Show that the equation $f^{-1}(x) = g^{-1}(x)$ has no solutions. You must show your working.

[3 marks]

Total for D3: 8 marks

D4 $f(x) = 2x^2 - a$ for $x \geq 0$, where a is a positive constant.

[4 marks]

$f^{-1}(3a) = 4$

Work out the value of a . You must show your working.

Answer $a = \dots\dots\dots$

D5 $f(x) = \frac{8}{x-2}$ for $x > 2$

(a) State the range of f .

[1 mark]

Answer $\dots\dots\dots$

(b) Work out $f^{-1}(x)$.

[2 marks]

Answer $f^{-1}(x) = \dots\dots\dots$

(c) Solve $f(x) = f^{-1}(x)$. You must show your working.

[4 marks]

Answer $x = \dots\dots$

Total for D5: 7 marks

D6 The function f has inverse $f^{-1}(x) = 2 + \sqrt{x-1}$ for $x \geq 1$.

[3 marks]

Work out $f(x)$ and state the domain of f .

Answer $f(x) = \dots\dots\dots$ domain $\dots\dots\dots$

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $f(x) = \frac{ax+7}{x-3}$ for $x \neq 3$, where a is a constant. The inverse function f^{-1} is the same function as f . Work out the value of a . [4 marks]

Answer $a = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Find $f^{-1}(x)$ in terms of a , then compare it with $f(x)$.
- Hint 2** $x = \frac{ay+7}{y-3}$ gives $xy - 3x = ay + 7$, so
 $y(x-a) = 3x+7$.
- Hint 3** $f^{-1}(x) = \frac{3x+7}{x-a}$. This is $\frac{ax+7}{x-3}$ for every x only when
 $a = 3$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the inverse of a function by swapping x and y and rearranging. ☐ ☐ ☐

I can choose the right square root when a function is on a restricted domain. ☐ ☐ ☐

I can state the domain of f^{-1} as the range of f . ☐ ☐ ☐

I can solve equations that use f^{-1} , such as $f(x) = f^{-1}(x)$. ☐ ☐ ☐