



Linear and quadratic inequalities

About 80 minutes No calculator 72 marks

Date _____

A Do Now

Quick questions to start.

- | | |
|--|--|
| <p>A1 Needed today [2 marks]</p> <p>Factorise $3x^2 - 7x - 6$.</p> | <p>A2 GCSE Higher [2 marks]</p> <p>Solve $7 - 3x > 19$.</p> |
|--|--|

Answer

Answer

- | | |
|--|--|
| <p>A3 Last lesson [2 marks]</p> <p>Show that $(x - 2)$ is a factor of $x^3 + 4x^2 - 3x - 18$.</p> | <p>A4 From chapter 3 [2 marks]</p> <p>$f(x) = 2x - 5$ with domain $1 \leq x \leq 6$. Work out the range of $f(x)$.</p> |
|--|--|

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Solve $3(x - 4) > 7x + 8$.

Hint Work as for an equation. If you divide by a negative number, the inequality sign turns round.

- B1** Solve $2(3 - 2x) \leq 5x - 21$. [3 marks]

Answer

EXAMPLE 2

Solve $-7 < 5 - 3x \leq 11$. Write down the integers that satisfy it.

Hint Do the same to all three parts. Dividing by -3 turns both signs round, so rewrite it with the smaller number first.

- B2** Solve $9 > 4 - 5x \geq -16$. Write down the integers that satisfy it. [3 marks]

Answer ; integers

EXAMPLE 3

$-1 \leq a \leq 5$ and $2 \leq b \leq 6$. Work out an inequality for $a - b$ and an inequality for a^2 .

Hint $a - b$ is least when a is least and b is greatest.
For a^2 : a can be 0, and the square of a negative number is positive.

B3 $-4 < x < 3$ and $1 < y < 5$. [4 marks]

Work out an inequality for $2x - y$ and an inequality for x^2 .

Answer $\dots\dots < 2x - y < \dots\dots$ $\dots\dots x^2 < \dots\dots$

EXAMPLE 4

Solve (i) $x^2 - 2x - 15 \leq 0$ and (ii) $x^2 - 2x - 15 > 0$.

Hint Factorise to find the critical values. Sketch the U-shaped graph: it is below the axis BETWEEN them and above the axis OUTSIDE them.

B4 Solve $2x^2 + x - 6 > 0$. [3 marks]

Answer $\dots\dots\dots$

EXAMPLE 5

Solve $14 - 5x > x^2$.

Hint Collect everything on the side where x^2 is positive first: $0 > x^2 + 5x - 14$.

B5 Solve $6 + x - 2x^2 < 0$. [4 marks]

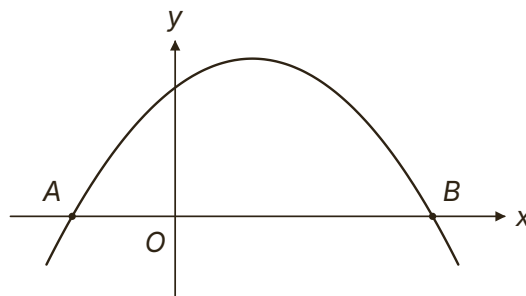
Answer $\dots\dots\dots$

C1 Solve $\frac{3x-2}{4} \geq x-3$.

[3 marks]

Answer

C2 The sketch shows the graph of $y = 10 + 3x - x^2$. It crosses the x -axis at A and B .



Not drawn accurately

(a) Work out the coordinates of A and B . [2 marks]

Answer $A(\dots, \dots)$ $B(\dots, \dots)$

(b) Hence solve $10 + 3x - x^2 < 0$. [2 marks]

Answer

Total for C2: 4 marks

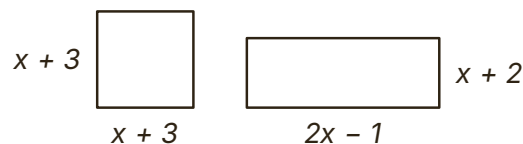
C3 $a < -2$ and $b > 3$. For each statement, write ALWAYS TRUE, SOMETIMES TRUE or NEVER TRUE.

[4 marks]

- (i) $ab < -6$ (ii) $a + b > 0$
 (iii) $a^2 > 4$ (iv) $b - a < 5$

C4 The perimeter of the rectangle is greater than the perimeter of the square. All lengths are in centimetres. Work out the values of x for which this is true.

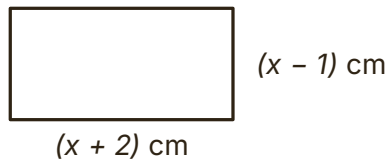
[3 marks]



Not drawn accurately

Answer

- C5** A rectangle measures $(x + 2)$ cm by $(x - 1)$ cm. Its area is less than 40 cm^2 . Work out the values of x for which this is true. [4 marks]



Not drawn accurately

- C6** How many integers satisfy $x^2 - 2x - 24 \leq 0$? [3 marks]

Answer

Answer

D Exam-style questions

AQA exam style

- D1** Circle the solution of $9 - x^2 < 0$ [1 mark]
- $-3 < x < 3$ $x < -3$ or $x > 3$ $x > 3$ $x < -9$ or $x > 9$

- D2** Max says, "To solve $x^2 \geq 6x$, I divide both sides by x . So the solution is $x \geq 6$."

Max is wrong.

- (a)** Give a value of x that satisfies $x^2 \geq 6x$ but is not in Max's answer. [1 mark]

Answer $x = \dots$

- (b)** Solve $x^2 \geq 6x$ [3 marks]

Answer

Total for D2: 4 marks

D3 A ball is thrown upwards. After t seconds its height is h metres, where

$$h = 35t - 5t^2$$

(a) Work out the values of t for which the ball is more than 50 m high.

[3 marks]

Answer

(b) For how many seconds is the ball more than 50 m high?

[1 mark]

Answer seconds

Total for D3: 4 marks

D4 Work out the values of x that satisfy both

[5 marks]

$$2x + 1 > 7 \quad \text{and} \quad x^2 - 4x - 21 < 0$$

You must show your working.

Answer

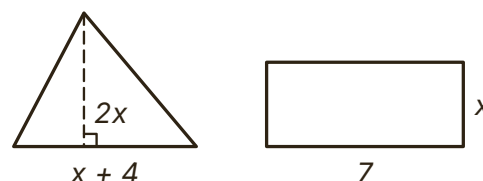
D5 n is an integer and $17n > 6(3n - 4)$. Work out the greatest possible value of n .

[3 marks]

Answer

D6 The area of the triangle is less than the area of the rectangle. All lengths are in centimetres. Work out the values of x for which this is true.

[4 marks]



Answer

Not drawn accurately

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The equation $x^2 + (k - 1)x + k + 2 = 0$ has two different real roots. Work out the possible values of k . [5 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Two different real roots means the discriminant $b^2 - 4ac$ is positive.
- Hint 2** $(k - 1)^2 - 4(k + 2) > 0$. Expand and collect into a quadratic inequality in k .
- Hint 3** $k^2 - 6k - 7 > 0$, so $(k - 7)(k + 1) > 0$. The critical values are -1 and 7 ; choose the outer regions.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can solve linear and double inequalities. ☐ ☐ ☐
- I can combine two ranges of values. ☐ ☐ ☐
- I can solve a quadratic inequality. ☐ ☐ ☐
- I can form an inequality from a problem. ☐ ☐ ☐