



Manipulating surds

About 75 minutes No calculator 63 marks

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier lessons.

- A1** Needed today [2 marks]

Write $\sqrt{72}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible.

A2 Needed today [2 marks]

Expand and simplify $(2x + 7)(2x - 7)$.

Answer

Answer

- A3** Last lesson [2 marks]

Work out the coefficient of x^2 in the expansion of $(1 - 2x)^5$.

A4 From chapter 1 [2 marks]

Solve $\frac{3x + 1}{4} = \frac{x + 5}{2}$.

Answer

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Simplify $2\sqrt{45} - \sqrt{20} + \sqrt{80}$.

Hint Write each root with the largest square factor taken out. Then they are all multiples of $\sqrt{5}$.

- B1** Simplify $3\sqrt{12} + \sqrt{75} - 2\sqrt{27}$. [3 marks]

Answer

EXAMPLE 2

Expand and simplify $(3 - 2\sqrt{5})(1 + \sqrt{5})$.

Hint Four products, as with $(a + b)(c + d)$. Remember $\sqrt{5} \times \sqrt{5} = 5$.

- B2** Expand and simplify $(\sqrt{7} - 2)(3\sqrt{7} + 4)$. [2 marks]

Answer

EXAMPLE 3

Rationalise the denominator of $\frac{5 + \sqrt{3}}{2 - \sqrt{3}}$. Give your answer in the form $a + b\sqrt{3}$.

Hint Multiply the top and the bottom by $2 + \sqrt{3}$, the bottom with its sign changed. The bottom becomes $4 - 3$.

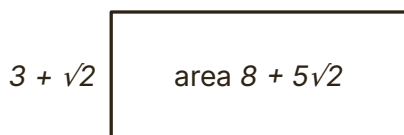
EXAMPLE 4

Solve $x\sqrt{12} + 5 = x\sqrt{3} + 17$. Give your answer in the form $a\sqrt{3}$.

Hint $\sqrt{12} = 2\sqrt{3}$. Collect the x terms on one side, then rationalise.

EXAMPLE 5

A rectangle is $(3 + \sqrt{2})$ cm wide. Its area is $(8 + 5\sqrt{2})$ cm². Work out its length. Give your answer in the form $a + b\sqrt{2}$.



Not drawn accurately

Hint Length = area $\div$ width. Write it as a fraction and rationalise with $3 - \sqrt{2}$.

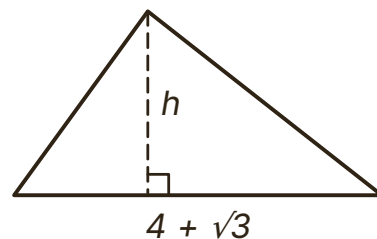
B3 Write $\frac{7 + 3\sqrt{2}}{3 + 2\sqrt{2}}$ in the form $a + b\sqrt{2}$, where a and b are integers. [3 marks]

Answer

B4 Solve $x\sqrt{27} - 4 = x\sqrt{12} + 11$. [3 marks]
Give your answer in the form $a\sqrt{3}$.

Answer $x = \dots\dots$

B5 A triangle has a base of $(4 + \sqrt{3})$ cm. Its area is $(7 + 5\sqrt{3})$ cm². Work out its perpendicular height h . Give your answer in the form $a + b\sqrt{3}$. [3 marks]



Not drawn accurately

Answer $h = \dots\dots$ cm

C Practice

Level 2

C1 Simplify

(a) $\sqrt{180}$ [1 mark]

Answer

(b) $\sqrt{6} \times \sqrt{15}$ [2 marks]

Answer

(c) $\frac{\sqrt{84}}{\sqrt{7}}$ [1 mark]

Answer

Total for C1: 4 marks

C2 Rationalise the denominator and simplify.

(a) $\frac{10}{\sqrt{5}}$ [1 mark]

Answer

(b) $\frac{9}{2\sqrt{3}}$ [2 marks]

Answer

Total for C2: 3 marks

C3 Write $\frac{1}{3-\sqrt{5}} + \frac{1}{3+\sqrt{5}}$ as a [3 marks]
single fraction in its simplest form.

Answer

C4 Use the expansion of $(a+b)^3$ to [3 marks]
write $(\sqrt{3}-1)^3$ in the form
 $p+q\sqrt{3}$.

Answer

C5 Expand and simplify $(\sqrt{6}+\sqrt{2})^2$. [2 marks]
Give your answer in the form
 $a+b\sqrt{3}$.

Answer

C6 Show that $(2\sqrt{5}-3)(2\sqrt{5}+3)$ is [2 marks]
an integer.**D Exam-style questions**

AQA exam style

D1 Kai says, " $\sqrt{12} + \sqrt{27} = \sqrt{39}$." [3 marks]Explain why Kai is wrong. Write $\sqrt{12} + \sqrt{27}$ in its simplest form.

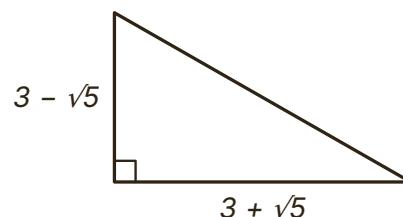
Answer

- D2** Rationalise and simplify $\frac{2\sqrt{3}+1}{\sqrt{3}-1}$. Give your answer in the form $\frac{a+b\sqrt{3}}{2}$, where a and b are integers. [4 marks]

Answer

- D3** The diagram shows a right-angled triangle. The two shorter sides are $(3 + \sqrt{5})$ cm and $(3 - \sqrt{5})$ cm.

- (a) Show that the longest side is $2\sqrt{7}$ cm. [2 marks]



Not drawn accurately

- (b) Work out the area of the triangle. [2 marks]

Answer cm²

- (c) Hence write down the perimeter of the triangle. [1 mark]

Answer cm

Total for D3: 5 marks

- D4** Solve $2x + \sqrt{3} = x\sqrt{3} + 5$. Give your answer in the form $a + b\sqrt{3}$, where a and b are integers. [4 marks]

Answer $x = \dots$

- D5** Which of these is equal to $\frac{\sqrt{98}}{\sqrt{2}}$? Circle your answer. $7\sqrt{2}$ 49 7 $\sqrt{96}$ [1 mark]

- D6** Show that $(3\sqrt{2} - 2)^2 + (3\sqrt{2} + 2)^2$ is an integer. You must show your working. [2 marks]

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Show that $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \cdots + \frac{1}{\sqrt{15} + \sqrt{16}} = 3$ [5 marks]

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Rationalise one of the fractions, say $\frac{1}{\sqrt{2} + \sqrt{3}}$, and look at what you get.
- Hint 2** Multiply the top and bottom of $\frac{1}{\sqrt{n} + \sqrt{n+1}}$ by $\sqrt{n+1} - \sqrt{n}$. The bottom becomes $(n+1) - n = 1$, so each fraction is $\sqrt{n+1} - \sqrt{n}$.
- Hint 3** The sum is $(\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + \cdots + (\sqrt{16} - \sqrt{15})$. Every root except $\sqrt{16}$ and $\sqrt{1}$ cancels, leaving $4 - 1 = 3$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can simplify, add and multiply surds. ☐ ☐ ☐
- I can rationalise a denominator with one or two terms. ☐ ☐ ☐
- I can solve an equation with surds and give an exact answer. ☐ ☐ ☐
- I can use surds in area and Pythagoras problems. ☐ ☐ ☐