

Mensuration, Pythagoras' theorem and angle facts

About 80 minutes No calculator 61 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-mensuration-pythagoras-theorem-and-angle-facts/

Date _____

A Do Now

The first two are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

A right-angled triangle has shorter sides 4 cm and 6 cm. Work out the length of the hypotenuse. Give your answer as a surd in its simplest form.

A2 Needed today [2 marks]

Work out the size of each interior angle of a regular polygon with 18 sides.

Answer

Answer

- A3** Last lesson [2 marks]

The point $P(5, -2)$ lies on the circle $x^2 + y^2 = 29$. Work out the gradient of the tangent to the circle at P .

A4 From chapter 2 [2 marks]

Rearrange $V = \frac{1}{3}\pi r^2 h$ to make r the subject, where $r > 0$.

Answer

Answer

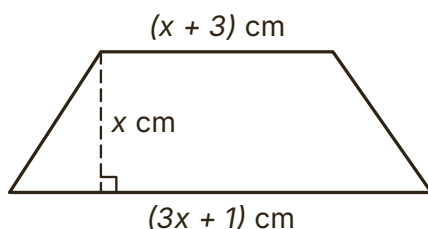
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

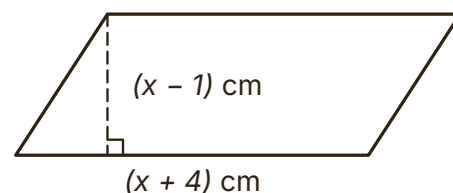
The trapezium has an area of 60 cm^2 . Work out the value of x .



Not drawn accurately

Hint Area = $\frac{1}{2}(a + b)h$. Set it equal to 60.

- B1** The parallelogram has an area of 50 cm^2 . Work out the value of x . [3 marks]

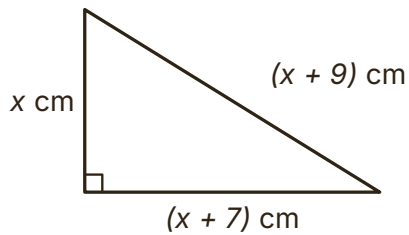


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Answer $x = \dots\dots$

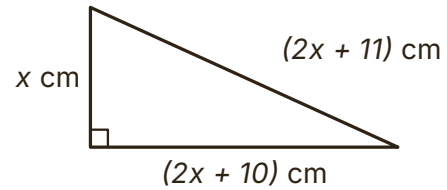
EXAMPLE 2

The sides of a right-angled triangle are x cm, $(x + 7)$ cm and $(x + 9)$ cm. Work out the perimeter of the triangle.



Not drawn accurately

Hint The longest side is the hypotenuse. Use Pythagoras' theorem, expand and collect terms.

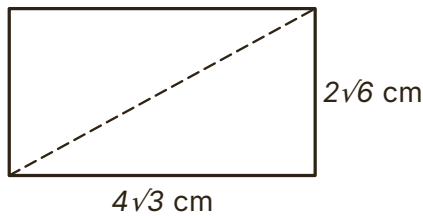
B2 The sides of a right-angled triangle are x cm, $(2x + 10)$ cm and $(2x + 11)$ cm. Work out the area of the triangle. [4 marks]

Not drawn accurately

Answer cm^2

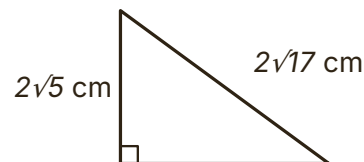
EXAMPLE 3

A rectangle measures $4\sqrt{3}$ cm by $2\sqrt{6}$ cm. Work out the length of its diagonal. Give your answer as a surd in its simplest form.



Not drawn accurately

Hint $(4\sqrt{3})^2 = 16 \times 3$. Add the two squares, then simplify the square root.

B3 A right-angled triangle has hypotenuse $2\sqrt{17}$ cm and one shorter side $2\sqrt{5}$ cm. Work out the area of the triangle. Give your answer in the form $a\sqrt{b}$, where a and b are integers. [3 marks]

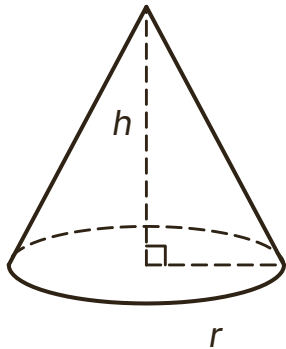
Not drawn accurately

Answer cm^2

EXAMPLE 4

A cone has a volume of $108\pi \text{ cm}^3$. Its height and base radius are in the ratio $h : r = 3 : 2$. Work out the base radius.

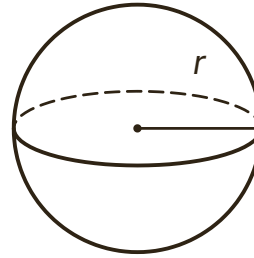
$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$



Hint Write h in terms of r : $h = \frac{3}{2}r$. Then the volume is a multiple of r^3 .

B4 A sphere has a surface area of $196\pi \text{ cm}^2$. Work out its volume. Give your answer as a multiple of π . [3 marks]

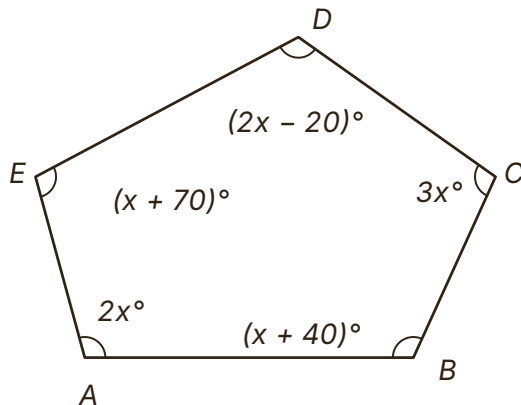
$$\begin{aligned} \text{Surface area of a sphere} &= 4\pi r^2 \\ \text{Volume of a sphere} &= \frac{4}{3}\pi r^3 \end{aligned}$$



Answer cm^3

EXAMPLE 5

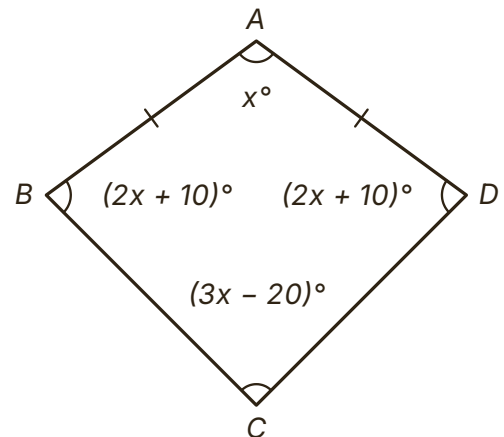
The pentagon $ABCDE$ has the angles shown. Work out the size of the largest angle.



Not drawn accurately

Hint The angles of a pentagon add up to $(5 - 2) \times 180 = 540^\circ$.

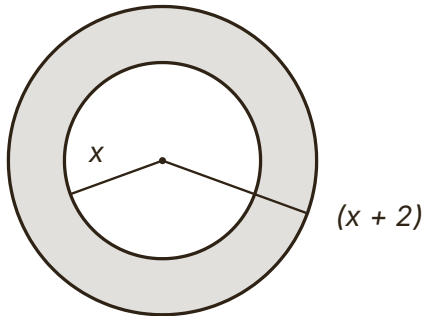
B5 $ABCD$ is a kite with $AB = AD$. Work out the size of angle BCD . [3 marks]



Not drawn accurately

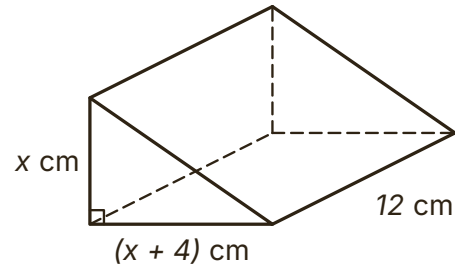
Answer $^\circ$

- C1** A washer is the region between two circles with the same centre. The radii are x cm and $(x + 2)$ cm. The area of the washer is 36π cm². Work out the value of x . [3 marks]



Not drawn accurately

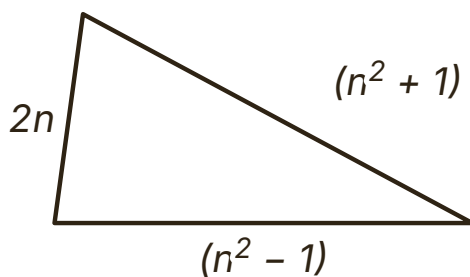
- C2** A prism is 12 cm long. Its cross-section is a right-angled triangle with shorter sides x cm and $(x + 4)$ cm. The volume of the prism is 270 cm³. Work out the value of x . [3 marks]



Not drawn accurately

Answer $x = \dots\dots$

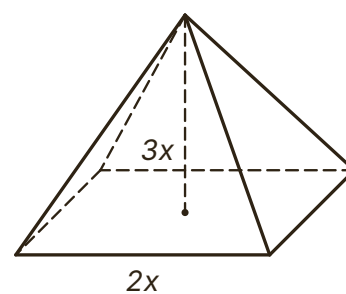
- C3** A triangle has sides of length $(n^2 - 1)$, $2n$ and $(n^2 + 1)$, where $n > 1$. Prove that the triangle is right-angled. [3 marks]



Not drawn accurately

- C4** A pyramid has a square base of side $2x$ cm and a vertical height of $3x$ cm. Its volume is 108 cm³. Work out the value of x . [3 marks]

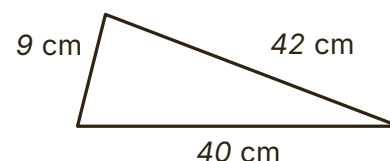
Volume of a pyramid = $\frac{1}{3} \times$ base area $\times$ height



Not drawn accurately

Answer $x = \dots\dots$

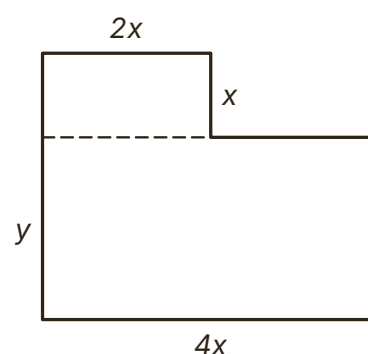
- D1** Mia says, "A triangle with sides 9 cm, 40 cm and 42 cm is right-angled." [2 marks]
Show that Mia is wrong.



Not drawn accurately

- D2** This shape is made from two rectangles. All lengths are in centimetres.
The perimeter of the shape is 72 cm.

- (a) Show that $y = 36 - 5x$. [2 marks]



Not drawn accurately

- (b) The area of the shape is $A \text{ cm}^2$. Show that $A = 144x - 18x^2$. [2 marks]

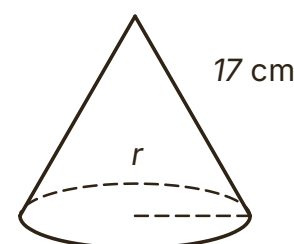
- (c) Write $144x - 18x^2$ in the form $p - q(x - r)^2$. Hence work out the greatest possible area of the shape. [3 marks]

Answer $(x - \dots\dots)^2$ greatest area cm^2

Total for D2: 7 marks

- D3** A cone has a slant height of 17 cm. Its curved surface area is $136\pi \text{ cm}^2$. Work out the volume of the cone. Give your answer as a multiple of π . [4 marks]

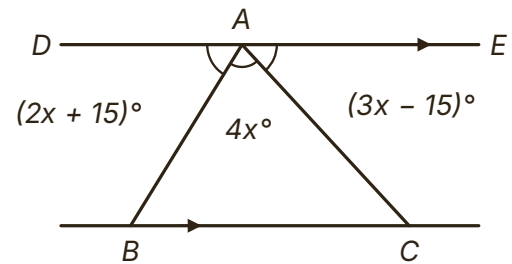
Curved surface area of a cone = πrl Volume of a cone = $\frac{1}{3}\pi r^2 h$



Answer cm^3

Not drawn accurately

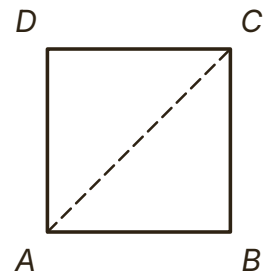
- D4** DAE is a straight line parallel to BC . Work out the size of angle ACB . Give a reason for each step of your working. [4 marks]



Answer°

Not drawn accurately

- D5** $ABCD$ is a square. The diagonal AC is $(4 + 2\sqrt{2})$ cm. Work out the area of $ABCD$. [3 marks]
Give your answer in the form $a + b\sqrt{2}$, where a and b are integers.



Answer + $\sqrt{2}$ cm²

Not drawn accurately

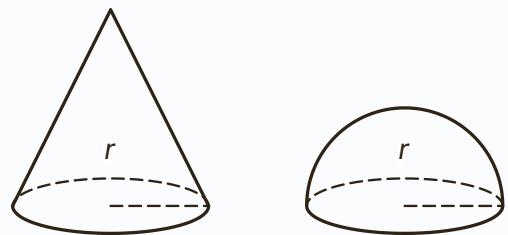
E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A solid cone and a solid hemisphere have the same base radius r . Their total surface areas are equal. Work out the ratio volume of the cone : volume of the hemisphere. Give your answer in the form $\sqrt{a} : b$, where a and b are integers. [5 marks]

Curved surface area of a cone = πrl Surface area of a sphere = $4\pi r^2$



Answer $\sqrt{\dots} : \dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1 Hint 1** Write both total surface areas using r and the slant height l .
Hint 2 Hemisphere: $3\pi r^2$. Cone: $\pi rl + \pi r^2$. So $l = 2r$.
Hint 3 $h = \sqrt{3}r$. Volumes $\frac{1}{3}\sqrt{3}\pi r^3$ and $\frac{2}{3}\pi r^3$; divide both by $\frac{1}{3}\pi r^3$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use area and volume formulae with algebra and π . ○ ○ ○

I can use Pythagoras' theorem with algebra and surds. ○ ○ ○

I can use angle facts to form and solve equations. ○ ○ ○