



Multiplying matrices and the identity matrix

About 75 minutes No calculator 74 marks

Date _____

A Do Now

The first questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Solve the simultaneous equations $4x + 3y = 7$ and $2x - y = 11$.

A2 Needed today [2 marks]

Solve $x^2 - 2x = 15$.

Answer

Answer $x = \dots\dots$ $y = \dots\dots$

- A3** From chapter 8 [2 marks]

Work out the gradient of the curve $y = 2x^3 - 5x$ at the point where $x = -1$.

A4 Last lesson [2 marks]

Work out the coordinates of the stationary point on the curve $y = 3x^2 - 12x + 5$.

Answer

Answer $(\dots\dots, \dots\dots)$

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$. Work out $2\mathbf{A}$ and $\mathbf{A}\mathbf{v}$.

Hint To multiply by a number, multiply every entry. For $\mathbf{A}\mathbf{v}$, multiply each row of $\mathbf{A}$ by the column.

B1 $\mathbf{A} = \begin{pmatrix} -1 & 3 \\ 4 & 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$. [3 marks]
Work out $3\mathbf{A}$ and $\mathbf{A}\mathbf{v}$.

EXAMPLE 2

$\mathbf{P} = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 1 & -2 \\ 5 & 3 \end{pmatrix}$. Work out $\mathbf{PQ}$ and $\mathbf{QP}$.

Hint Row i of the first matrix times column j of the second gives the entry in row i , column j .

B2

$$\mathbf{M} = \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix} \text{ and}$$

[4 marks]

$$\mathbf{N} = \begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix}. \text{ Work out } \mathbf{MN} \text{ and } \mathbf{NM}.$$

EXAMPLE 3

$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$. (a) Work out $\mathbf{A}^2$. (b) Explain why $\mathbf{vA}$ cannot be worked out.

Hint $\mathbf{A}^2$ means $\mathbf{AA}$. A product needs the number of columns of the first to equal the number of rows of the second.

B3

$$\mathbf{B} = \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix} \text{ and } \mathbf{w} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}. \quad [4 \text{ marks}]$$

(a) Work out $\mathbf{B}^2$. (b) One of $\mathbf{Bw}$ and $\mathbf{wB}$ can be worked out. Work it out.

EXAMPLE 4

Given that $\begin{pmatrix} 2 & a \\ 1 & 3 \end{pmatrix} \begin{pmatrix} b & 4 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 4 \\ -1 & 10 \end{pmatrix}$, work out the values of a and b .

Hint Multiply out the left-hand side in terms of a and b . Then equate the entries that hold only one unknown first.

B4

Given that [4 marks]

$$\begin{pmatrix} 3 & p \\ -1 & 2 \end{pmatrix} \begin{pmatrix} q & 1 \\ 2 & -4 \end{pmatrix} = \begin{pmatrix} 12 & -9 \\ 2 & -9 \end{pmatrix},$$

work out the values of p and q .

Answer $p = \dots \dots \dots$ $q = \dots \dots \dots$

EXAMPLE 5

The identity matrix is $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Given that $\begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \mathbf{I}$, work out the values of a , b , c and d .

Hint For any 2×2 matrix $\mathbf{A}$, $\mathbf{AI} = \mathbf{IA} = \mathbf{A}$. Equate the first column to get two equations in a and c , then the second column for b and d .

B5

Given that [5 marks]

$$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \mathbf{I}, \text{ where } \mathbf{I} \text{ is the identity matrix, work out the values of } p, q, r \text{ and } s.$$

Answer $p = \dots$ $q = \dots$ $r = \dots$ $s = \dots$

C Practice

Level 2

- C1** $\mathbf{A} = \begin{pmatrix} 1 & k \\ 2 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 4 \\ 4 & 7 \end{pmatrix}$. [4 marks] Work out the value of k for which $\mathbf{AB} = \mathbf{BA}$.
- C2** $\mathbf{M} = \begin{pmatrix} x & 5 \\ 1 & x \end{pmatrix}$. Given that $\mathbf{M}^2 = \begin{pmatrix} 14 & -30 \\ -6 & 14 \end{pmatrix}$, work out the value of x . [4 marks]

Answer $k = \dots\dots$ Answer $x = \dots\dots$

- C3** Given that $\begin{pmatrix} 3 & -2 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$, work out the values of x and y . [4 marks]
- C4** $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & -3 \end{pmatrix}$. Show that $\mathbf{A}^2 = k\mathbf{I}$, where k is a number, and state the value of k . Hence work out $\mathbf{A}^4$. [4 marks]

Answer $x = \dots\dots$ $y = \dots\dots$ **D Exam-style questions**

AQA exam style

- D1** Leo works out $\begin{pmatrix} 3 & 2 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ -1 & 5 \end{pmatrix}$. He writes $\begin{pmatrix} 12 & 2 \\ -1 & -10 \end{pmatrix}$. [3 marks]

Explain Leo's mistake. Work out the correct answer.

- D2** $\begin{pmatrix} 2 & c \\ 0 & 5 \end{pmatrix} \begin{pmatrix} a & -3 \\ 0 & 2 \end{pmatrix} = k\mathbf{I}$, where k is a constant and $\mathbf{I}$ is the identity matrix. Work out the values of a , c and k . [4 marks]

Answer $a = \dots\dots$ $c = \dots\dots$ $k = \dots\dots$

D3 $\mathbf{M} = \begin{pmatrix} k & 3 \\ 2 & 1 \end{pmatrix}$, where k is a positive number.

(a) Work out $\mathbf{M}^2$. Give your answer in terms of k .

[2 marks]

(b) The top left entry of $\mathbf{M}^2$ is 22. Work out the value of k .

[2 marks]

Answer $k = \dots\dots$

(c) Use your value of k to show that $\mathbf{M}^2 - 5\mathbf{M} = 2\mathbf{I}$.

[2 marks]

(d) Hence work out the matrix $\mathbf{N}$ such that $\mathbf{MN} = \mathbf{I}$.

[2 marks]

Total for D3: 8 marks

D4 Given that $\begin{pmatrix} x & 1 \\ 2 & x \end{pmatrix} \begin{pmatrix} x \\ 3 \end{pmatrix} = \begin{pmatrix} 19 \\ y \end{pmatrix}$, work out the two possible pairs of values of x and y .

[4 marks]

Answer $x = \dots\dots$, $y = \dots\dots$ or $x = \dots\dots$, $y = \dots\dots$

D5 $\mathbf{M}$ is a 2×2 matrix such that $\begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} \mathbf{M} = \begin{pmatrix} 4 & 1 \\ 11 & 2 \end{pmatrix}$. Work out $\mathbf{M}$.

[5 marks]

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 **M** is a 2×2 matrix. Every entry of **M** is a positive whole number. $\mathbf{M}^2 = \begin{pmatrix} 7 & 3 \\ 9 & 4 \end{pmatrix}$. [6 marks]
Work out **M**.

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Let $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and work out $\mathbf{M}^2$ in terms of a, b, c and d .
- Hint 2** The two diagonal entries give $a^2 + bc = 7$ and $d^2 + bc = 4$. Subtract them: $a^2 - d^2 = 3$, so $(a - d)(a + d) = 3$.
- Hint 3** With whole numbers, $a + d = 3$ and $a - d = 1$, so $a = 2$ and $d = 1$. Then $3b = 3$ and $3c = 9$, so $\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can multiply a 2 by 2 matrix by a number, by a column and by another 2 by 2 matrix. ☐ ☐ ☒
- I know that $\mathbf{AB}$ and $\mathbf{BA}$ are usually different, and when a product cannot be worked out. ☐ ☐ ☒
- I can find unknown entries by equating elements, including when the answer is $\mathbf{I}$. ☐ ☐ ☒
- I can find a matrix **M** with $\mathbf{AM} = \mathbf{I}$. ☐ ☐ ☒