



Ratio, linear equations, the binomial expansion, surds and counting

About 90 minutes No calculator
106 marks

Date _____

A Do Now

These bring back the GCSE Higher work that this chapter is built on.

A1 Needed today [2 marks]
Expand and simplify $(2x - 3)(x + 5)$.

Answer

A2 GCSE Higher [2 marks]
Simplify $\sqrt{32} + \sqrt{200}$.

Answer

A3 GCSE Higher [2 marks]
Work out $2\frac{2}{3} \times 1\frac{1}{8}$.

Answer

A4 GCSE Higher [2 marks]
Solve $4(2x - 1) = 3(x + 7)$.

Answer

Simplify $(3a^2b)^3$.

Answer

B Examples, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

The coefficient of x^2 in the expansion of $(1 + kx)^5$ is 90, where k is positive.

Work out the value of k . Hence work out the coefficient of x^3 .

Hint The x^2 term is $\binom{5}{2}(kx)^2 = 10k^2x^2$. Set $10k^2 = 90$.

B1 The coefficient of x^2 in the expansion of $(3 + kx)^4$ is 216, where k is negative. [4 marks]

Work out the value of k .
Hence work out the coefficient of x^3 .

Answer $k = \dots\dots$ coefficient $\dots\dots$

EXAMPLE 2

Solve $x\sqrt{2} + 10 = 3x + \sqrt{2}$.

Give your answer in the form $a + b\sqrt{2}$, where a and b are integers.

Hint Collect the x terms on one side and factorise: $x(\sqrt{2} - 3) = \sqrt{2} - 10$. Then rationalise.

B2 Solve $x\sqrt{5} + 13 = 3x + 3\sqrt{5}$. [4 marks]

Give your answer in the form $a + b\sqrt{5}$, where a and b are integers.

Answer $x = \dots\dots\dots$

EXAMPLE 3

Four-digit numbers are made from these cards. Each card is used at most once.

How many of the numbers are even and greater than 4000?

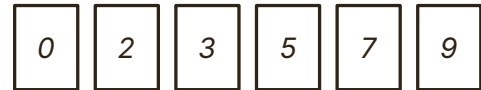


Hint Fill the two places with rules first: the first digit is 4, 6 or 8, and the last digit is even. Count each choice of first digit on its own.

B3 Three-digit numbers are [3 marks]

made from these cards. Each card is used at most once.

How many of the numbers are odd and less than 600?



Answer

EXAMPLE 4

A bag holds red and blue counters in the ratio 3 : 5. Then 12 red counters are added and 4 blue counters are taken out. The ratio of red to blue is now 7 : 9.

Work out how many counters are in the bag now.

Hint Call the counters $3k$ red and $5k$ blue. Then $\frac{3k + 12}{5k - 4} = \frac{7}{9}$.

B4 Ann's savings and Ben's [4 marks]

savings are in the ratio 4 : 3. Ann spends £6 and Ben saves another £9. The ratio of Ann's savings to Ben's savings is now 2 : 3.

How much does Ben have now?

Answer

C Practice

Level 2

Each question asks for something different. Show your working.

C1 Do not use a calculator.

(a) Write the ratio 2.4 kg : 750 g in its simplest form. [1 mark]

Answer

(b) Decrease £360 by 17.5%. [2 marks]

Answer

(c) Simplify fully $\frac{8a^2b^3}{6ab} \div \frac{2ab^2}{9b}$. [3 marks]

Answer

Total for C1: 6 marks

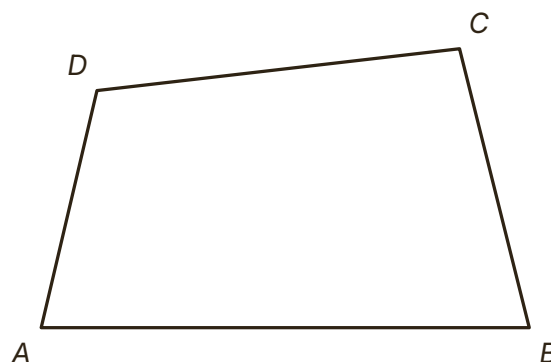
C2 Solve $\frac{3x-2}{4} + \frac{x+5}{6} = x-1$. [3 marks]

Answer

- C3** In the quadrilateral $ABCD$, angle $A = (2x + 10)^\circ$, angle $B = (3x - 5)^\circ$, angle $C = (x + 40)^\circ$ and angle $D = (4x - 15)^\circ$. [3 marks]

Work out the size of the largest angle.

Answer



Not drawn accurately

- C4** Expand and simplify $(2x - 1)(x + 3)(x - 4)$. [3 marks]

Answer

- C5** Expand $\left(2 - \frac{x}{2}\right)^4$. Give each term in its simplest form. [3 marks]

Answer

- C6** Write $\frac{14}{3 + \sqrt{2}}$ in the form $a + b\sqrt{2}$, where a and b are integers. [3 marks]

Answer

- C7** A code is three different letters chosen from A to H, followed by two digits from 0 to 9. The digits may be the same, but the last digit must be odd. [2 marks]
How many codes are possible?

Answer

- C8** Six different books are put in a row on a shelf. Two of the books are maths books, and they must be next to each other. [2 marks]
How many different orders are possible?

Answer

- C9** p is 35% of q , and $q : r = 8 : 5$. [3 marks]
Show that $25p = 14r$.

D Exam-style questions

AQA exam style

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** Liam makes three-digit numbers from the digits 0, 2, 5, 6, 8 and 9. He uses each digit at most once. He says, "There are $6 \times 5 \times 4 = 120$ numbers." [3 marks]

Liam is wrong. Explain why, and work out the correct number.

Answer

D2 $(x + 4)(ax - 3) - 2(x^2 + bx) \equiv 5x^2 + 15x - 12$

[4 marks]

Work out the values of a and b .

Answer $a = \dots\dots$ $b = \dots\dots$

D3 Do not use a calculator.

- (a) Use the binomial expansion to write $(1 + \sqrt{2})^4$ in the form $a + b\sqrt{2}$, where a [3 marks]
and b are integers.

Answer

- (b) Hence write down $(1 - \sqrt{2})^4$ in the form $a + b\sqrt{2}$. [1 mark]

Answer

- (c) Hence show that $\frac{1}{17 + 12\sqrt{2}} = 17 - 12\sqrt{2}$. [2 marks]

Total for D3: 6 marks

- D4** In 5 years' time Dan will be twice as old as Eve. Five years ago Dan was three [4 marks]
times as old as Eve.

Work out Dan's age now and Eve's age now. You must show your working.

Answer Dan Eve

- D5** At a show, the ratio of adults to children is 7 : 4. There are 30 more adults than children. 30% of the adults are men. [4 marks]

Work out the number of women at the show.

Answer

- D6** n is a positive integer. In the expansion of $(1 + 2x)^n$ the coefficient of x is 16. [4 marks]

Work out the value of n . Hence work out the coefficient of x^2 .

Answer $n = \dots\dots$ coefficient of $x^2 = \dots\dots$

- D7** A code is two letters followed by three digits. The letters are chosen from A, B, C, D and E, and the two letters are different. Each digit is from 1 to 9, and the digits may repeat.

- (a) How many codes are possible? [2 marks]

Answer

- (b) How many of the codes start with a vowel and end with an even digit? [2 marks]

Answer

Total for D7: 4 marks

- D8** Rationalise and simplify $\frac{\sqrt{6} + 4}{\sqrt{6} - 2}$. [4 marks]

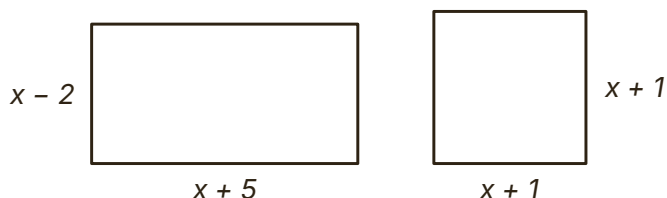
Give your answer in the form $a + b\sqrt{6}$, where a and b are integers.

Answer

- D9** A rectangle is $(x + 5)$ cm long and $(x - 2)$ cm wide. A square has sides of $(x + 1)$ cm. [5 marks]

The area of the rectangle is 4 cm^2 more than the area of the square.

Work out the value of x . Hence work out the perimeter of the rectangle.



Not drawn accurately

Answer $x = \dots\dots$ perimeter $\dots\dots$ cm

- D10** Which of these is equal to $(\sqrt{3})^5$? Circle your answer. [1 mark]

15 $9\sqrt{3}$ $3\sqrt{3}$ 243

Answer

E Extension

Stretch

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** How many four-digit numbers contain at least one digit 7? [3 marks]

Answer

E2 The expansion of $(1 + kx)(3 - x)^4$ has no term in x^2 . [4 marks]

Work out the value of k .

Answer

E3 a and b are integers and a is positive. [4 marks]

$$(a + b\sqrt{3})^2 = 37 - 20\sqrt{3}$$

Work out the values of a and b .

Answer $a = \dots\dots$ $b = \dots\dots$

E4 The digits of a two-digit number add up to 11. When the two digits are swapped, the number goes up by 45. [3 marks]

Work out the number.

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can simplify ratios, work with percentages and simplify algebraic expressions.	☐	☐	☐
I can form and solve linear equations, including ones with fractions.	☐	☐	☐
I can expand three brackets and use the binomial expansion to find a coefficient or an unknown.	☐	☐	☐
I can simplify surds, rationalise denominators and solve equations with surds.	☐	☐	☐
I can use the product rule to count arrangements with conditions.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

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- E1**
- Hint 1** It is easier to count the four-digit numbers with no 7 at all, and take them away from all the four-digit numbers.
- Hint 2** There are 9000 four-digit numbers. With no 7: the first digit has 8 choices (not 0 or 7) and each other digit 9.
- Hint 3** $8 \times 9 \times 9 \times 9 = 5832$ have no 7, so $9000 - 5832 = 3168$ have at least one.
-
- E2**
- Hint 1** Expand $(3 - x)^4$ as far as the x^2 term. Only two products in $(1 + kx)(3 - x)^4$ give x^2 .
- Hint 2** $(3 - x)^4 = 81 - 108x + 54x^2 - \dots$
- Hint 3** The x^2 terms are $1 \times 54x^2$ and $kx \times (-108x)$, so $54 - 108k = 0$ and $k = \frac{1}{2}$.
-
- E3**
- Hint 1** Expand the bracket and compare the whole-number parts and the $\sqrt{3}$ parts.
- Hint 2** $a^2 + 3b^2 = 37$ and $2ab = -20$.
- Hint 3** $ab = -10$. Try the pairs that multiply to -10 : $a = 5$, $b = -2$ gives $25 + 12 = 37$.
-
- E4**
- Hint 1** Call the tens digit t and the units digit u . The number is $10t + u$.
- Hint 2** $t + u = 11$ and $(10u + t) - (10t + u) = 45$.
- Hint 3** The second equation is $9(u - t) = 45$, so $u - t = 5$. With $t + u = 11$ this gives $u = 8$ and $t = 3$.
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