

Parallel and perpendicular lines, distance and midpoint

About 90 minutes No calculator 70 marks

Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-parallel-and-perpendicular-lines-distance-and-midpoint/



Date _____

A Do Now

The first two questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Work out the gradient of the line through the points $(-2, 5)$ and $(4, -7)$.

A2 Needed today [2 marks]

Write $\sqrt{80}$ in the form $a\sqrt{5}$, where a is an integer.

Answer $a = \dots\dots$

Answer

- A3** Last lesson [2 marks]

$x + y + z = 9$, $y = 2x$ and $z = x + 1$. Work out the values of x , y and z .

A4 From chapter 4 [2 marks]

Work out the value of $27^{-\frac{2}{3}}$.

Answer

Answer $x = \dots\dots$ $y = \dots\dots$ $z = \dots\dots$

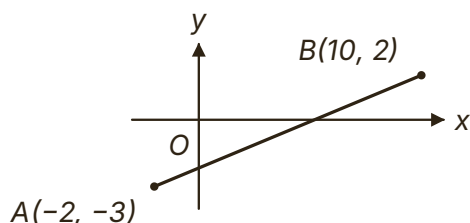
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A is the point $(-2, -3)$ and B is the point $(10, 2)$. Work out (a) the gradient of a line perpendicular to AB , (b) the length of AB , (c) the coordinates of the midpoint of AB .



Not drawn accurately

Hint Gradient of AB is the change in y over the change in x . A perpendicular gradient is -1 over it. For the length, use Pythagoras on the two changes.

- B1** P is $(1, 7)$ and Q is $(-5, 11)$. [5 marks]
- Work out the gradient of a line perpendicular to PQ , the exact length of PQ in its simplest form, and the midpoint of PQ .



Not drawn accurately

EXAMPLE 2

$M(3, -1)$ is the midpoint of the line AB . A is the point $(-4, 5)$. Work out the coordinates of B .

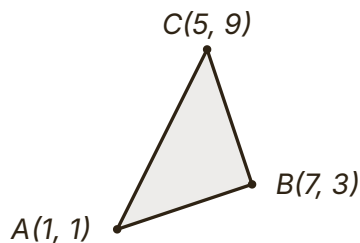
Hint To get from A to M you go some way in x and in y . Go the same again from M to reach B .

B2 $M(2, -3)$ is the midpoint of the line AB , where A is $(p, 4)$ and B is $(7, r)$. Work out the values of p and r . [2 marks]

Answer $p = \dots\dots$ $r = \dots\dots$

EXAMPLE 3

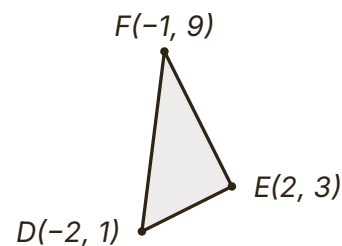
$A(1, 1)$, $B(7, 3)$ and $C(5, 9)$ are the vertices of a triangle. Show that the triangle is right-angled (a) by using gradients, (b) by using lengths.



Not drawn accurately

Hint Find the gradients of the three sides: two that multiply to -1 meet at a right angle. For lengths, test $a^2 + b^2 = c^2$ with the longest side as c .

B3 $D(-2, 1)$, $E(2, 3)$ and $F(-1, 9)$ are the vertices of a triangle. Show that the triangle is right-angled. [3 marks]



Not drawn accurately

EXAMPLE 4

A is $(1, 1)$, B is $(3, b)$ and C is $(7, 3)$. Angle $ABC = 90^\circ$. Work out the two possible values of b .

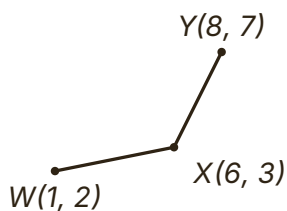
Hint Write the gradients of AB and BC in terms of b . A right angle at B means they multiply to -1 .

B4 X is $(-2, 3)$, Y is $(1, a)$ and Z is $(5, 2)$. Angle $XYZ = 90^\circ$. Work out the two possible values of a . [4 marks]

Answer $a = \dots\dots$ or $a = \dots\dots$

EXAMPLE 5

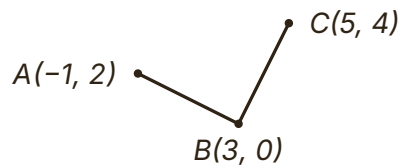
$WXYZ$ is a parallelogram. W is $(1, 2)$, X is $(6, 3)$ and Y is $(8, 7)$. Work out the coordinates of Z .



Not drawn accurately

Hint The diagonals of a parallelogram cut each other in half, so WY and XZ have the same midpoint.

B5 A is $(-1, 2)$, B is $(3, 0)$ and C is $(5, 4)$. [4 marks]
Show that angle $ABC = 90^\circ$. Hence work out the coordinates of D such that $ABCD$ is a rectangle.



Not drawn accurately

Answer $D = (\dots, \dots)$

c Practice

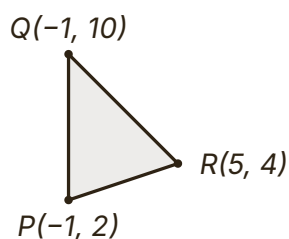
Level 2

C1 P is the point $(2a, b)$ and Q is the point $(6a, 7b)$, where a and b are positive. Write, in terms of a and b , (a) the gradient of PQ , (b) the length of PQ , (c) the midpoint of PQ . [4 marks]

C2 The distance between the points $A(3, k)$ and $B(-5, 1)$ is 17. Work out the two possible values of k . [3 marks]

Answer $k = \dots$ or $k = \dots$

- C3** $P(-1, 2)$, $Q(-1, 10)$ and $R(5, 4)$ [4 marks]
are the vertices of a triangle.
Work out the area of the triangle
and its exact perimeter, giving
the perimeter with each surd in
its simplest form.



Not drawn accurately

- C4** The points $P(-3, 1)$, $Q(1, q)$ and $R(9, 7)$ lie on a straight line. [2 marks]
Work out the value of q .

Answer $q = \dots\dots$

Answer area $\dots\dots$ perimeter $\dots\dots\dots$

- C5** A is $(0, 3)$, B is $(4, 5)$, C is $(8, 1)$ and D is $(0, -3)$. Show that [3 marks]
 $ABCD$ is a trapezium.

D Exam-style questions

AQA exam style

- D1** Work out the distance between the points $P(-4, 9)$ and $Q(3, -15)$. [2 marks]

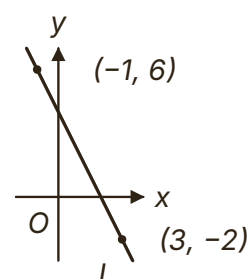
Answer $\dots\dots$ units

- D2** Ali says, "The line L has gradient -2 , so a line perpendicular to L has gradient 2 ."

The line L passes through the points $(-1, 6)$ and $(3, -2)$.

- (a) Work out the gradient of a line perpendicular to L . [2 marks]

Answer $\dots\dots\dots$



Not drawn accurately

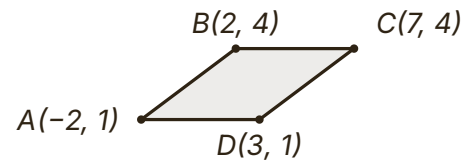
(b) Explain what Ali has done wrong.

[1 mark]

Total for D2: 3 marks

D3 A is $(-2, 1)$, B is $(2, 4)$, C is $(7, 4)$ and D is $(3, 1)$.

(a) Show that $ABCD$ is a rhombus. [3 marks]



Not drawn accurately

(b) Show that the diagonals AC and BD are perpendicular. [2 marks]

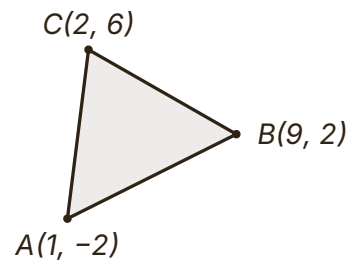
(c) Work out the area of $ABCD$. [2 marks]

Answer

Total for D3: 7 marks

D4 A is $(1, -2)$, B is $(9, 2)$ and C is $(2, 6)$.

(a) Show that triangle ABC is isosceles. [3 marks]



(b) M is the midpoint of AB . Work out the coordinates of M . [1 mark]

Not drawn accurately

Answer (.....,.....)

(c) Hence work out the area of triangle ABC . [3 marks]

Answer

Total for D4: 7 marks

- D5** The point $P(k, 2k)$ is 5 units from the point $Q(1, -3)$. Work out the two possible coordinates of P . You must show your working. [4 marks]

Answer (.....,.....) or (.....,.....)

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** $A(1, 1)$ and $B(5, 3)$ are two corners of a square. Work out every possible pair of positions for the other two corners. [5 marks]

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** AB could be a side of the square (on either side of it) or a diagonal. Use the step from A to B .
- Hint 2** From A to B is 4 across and 2 up. A step at right angles is 2 back and 4 up, or 2 across and 4 down.
- Hint 3** As a side: add $(-2, 4)$ to A and B , or add $(2, -4)$. As a diagonal: the centre is $(3, 2)$ and the half-diagonal $(2, 1)$ turned a right angle is $(-1, 2)$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can tell if two lines are parallel or perpendicular from their gradients. ☐ ☐ ☐
- I can work out the exact length and the midpoint of a line between two points. ☐ ☐ ☐
- I can find an unknown coordinate from a right angle, a distance or a midpoint. ☐ ☐ ☐
- I can prove facts about triangles and quadrilaterals using coordinates. ☐ ☐ ☐