



Problems in three dimensions

About 90 minutes Calculator allowed 63 marks

Date _____

A Do Now

Answer these before the lesson starts.

- A1** Needed today [2 marks]

A cuboid measures 8 cm by 9 cm by 12 cm. Work out the length of a diagonal from one corner to the opposite corner.

A2 Needed today [2 marks]

In a right-angled triangle, the side opposite angle θ is 7.2 cm and the other shorter side is 5 cm. Work out θ .

Answer cm

Answer $\theta = \dots\dots^\circ$

- A3** Last lesson [2 marks]

A triangle has sides of 5 cm, 7 cm and 9 cm. Work out the size of its largest angle.

A4 From chapter 7 [2 marks]

A triangle has sides of 11 cm and 14 cm with an angle of 35° between them. Work out its area.

Answer $^\circ$

Answer cm²

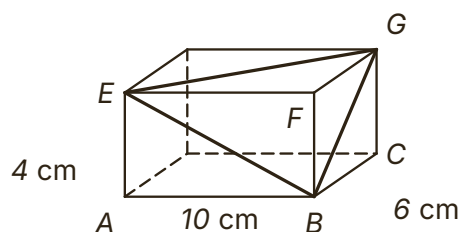
B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it. Give lengths and angles to 3 significant figures. In each cuboid, E , F and G are above A , B and C .

EXAMPLE 1

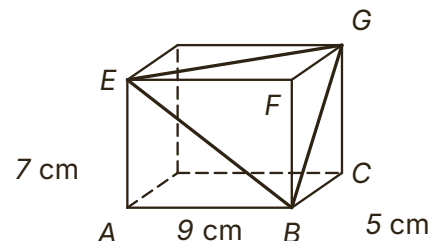
The diagram shows a cuboid. Work out the size of angle EBG in the triangle BEG .



Not drawn accurately

Hint Each side is a diagonal of a face: find all three by Pythagoras. Then use the cosine rule.

- B1** The diagram shows a cuboid. [4 marks]
Work out the size of angle BEG in the triangle BEG .

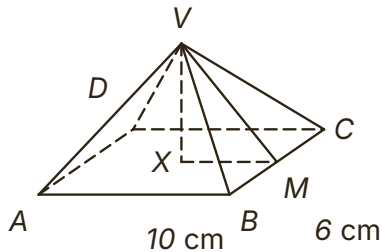


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Answer $^\circ$

EXAMPLE 2

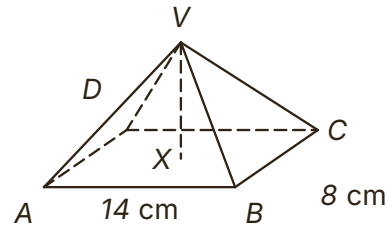
Pyramid $VABCD$ has a horizontal rectangular base. V is above its centre X and $VX = 8$ cm. M is the midpoint of BC . Work out the angle between the base and (a) the face VBC (b) the edge VB .



Not drawn accurately

Hint The planes meet along BC : use VM and XM , both at right angles to BC . XM is half of AB . For the edge, use XB .

- B2** Pyramid $VABCD$ has a horizontal rectangular base. V is above its centre X and $VX = 11$ cm. Work out the angle between the base and (a) the face VAB (b) the face VBC . [4 marks]

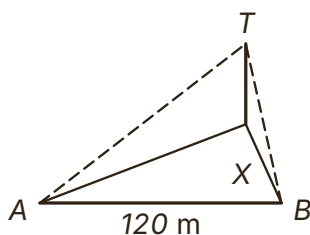


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Answer (a)° (b)°

EXAMPLE 3

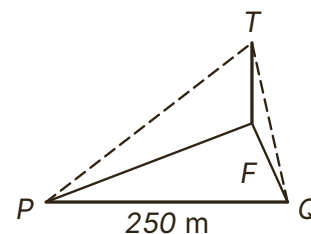
A level road AB is 120 m long. XT is a vertical mast with its foot X level with the road. Angle $XAB = 34^\circ$, angle $XBA = 58^\circ$ and the angle of elevation of T from A is 21° . Work out (a) the height XT (b) the angle of elevation of T from B .



Not drawn accurately

Hint On the ground, use the sine rule in triangle ABX . Each triangle with XT in it is right-angled at X .

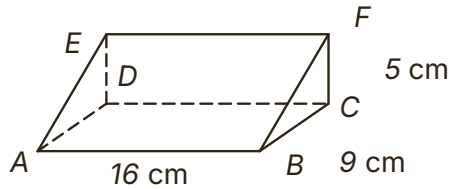
- B3** A level road PQ is 250 m long. FT is a vertical mast with its foot F level with the road. Angle $FPQ = 41^\circ$, angle $FQP = 63^\circ$ and the angle of elevation of T from Q is 18° . Work out (a) the height FT (b) the angle of elevation of T from P . [5 marks]



Not drawn accurately

Answer (a) m (b)°

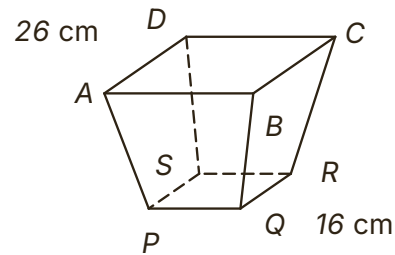
- C1** The diagram shows a wedge with a horizontal rectangular base $ABCD$ and a vertical rectangular face $CDEF$. Work out (a) angle FBC (b) the angle between AF and the base $ABCD$. [4 marks]



Not drawn accurately

Answer (a)° (b)°

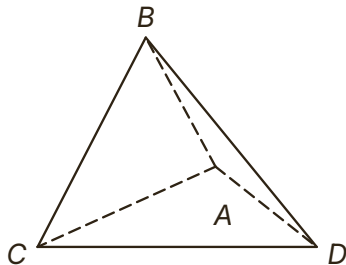
- C2** A basket is the shape of part of a square-based pyramid. Its top $ABCD$ is a square of side 26 cm and its base $PQRS$ is a square of side 16 cm. The top and the base are horizontal, with their centres on one vertical line, 30 cm apart. Work out (a) the length AP (b) the angle between AP and the base $PQRS$. [4 marks]



Not drawn accurately

Answer (a) cm (b)°

- C3** $ABCD$ is a tetrahedron. $AC = 6$ cm, $AD = 9$ cm and angle $CAD = 118^\circ$. In the face BCD , angle $BCD = 70^\circ$ and angle $BDC = 48^\circ$. Work out the length BC . [4 marks]



Not drawn accurately

Answer $BC =$ cm

D Exam-style questions

AQA exam style

- D1** Two lines in three dimensions do not meet and are not parallel. Which word describes them? Circle your answer. [1 mark]

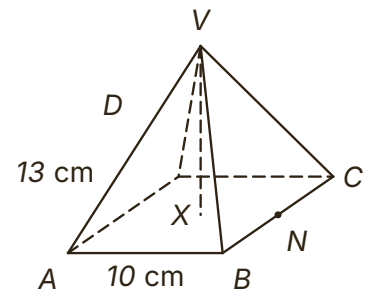
parallel skew perpendicular intersecting

- D2** Ella says, "The face VBC contains the edge VB , so the angle between the face VBC and the base is the same as the angle between the edge VB and the base."

$VABCD$ is a pyramid with a horizontal square base $ABCD$ of side 10 cm. V is directly above X , the centre of the base. $VA = VB = VC = VD = 13$ cm. N is the midpoint of BC .

- (a) Show that the height VX is $\sqrt{119}$ cm. [2 marks]

- (b) Work out the angle between the edge VA and the base. [2 marks]



Not drawn accurately

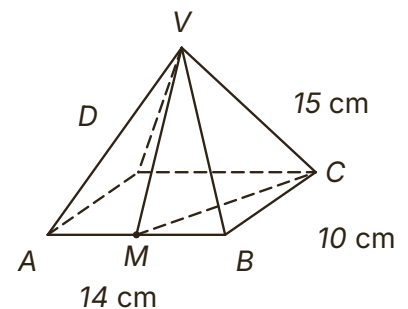
Answer°

- (c) Show that Ella is wrong. [3 marks]

Total for D2: 7 marks

- D3** $VABCD$ is a pyramid with a horizontal rectangular base $ABCD$. V is directly above the centre of the base. $AB = 14$ cm, $BC = 10$ cm and $VA = VB = VC = VD = 15$ cm. M is the midpoint of AB .

- (a) Work out the size of angle MVC . You must show your working. [5 marks]



Not drawn accurately

Answer°

(b) Work out the area of triangle MVC .

[2 marks]

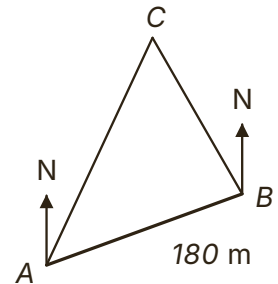
Answer cm^2

Total for D3: 7 marks

D4 A , B and C are points on horizontal ground. B is 180 m from A on a bearing of 070° . The bearing of C from A is 025° and the bearing of C from B is 330° . C is the foot of a vertical tower CT . The angle of elevation of T from A is 12° .

(a) Show that angle $ACB = 55^\circ$ and work out the distance AC .

[3 marks]



Answer $AC = \dots\dots$ m

(b) Work out the height of the tower.

[2 marks]

Not drawn accurately

Answer m

(c) Work out the angle of elevation of T from B .

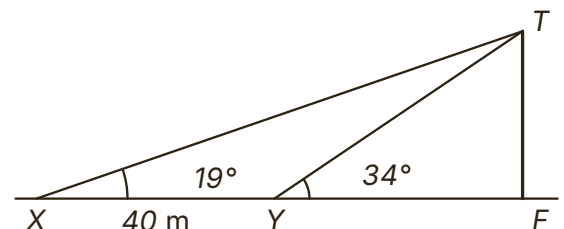
[2 marks]

Answer $^\circ$

Total for D4: 7 marks

D5 X and Y are 40 m apart on level ground, in line with the foot F of a vertical mast FT . From X , the angle of elevation of T is 19° . From Y , it is 34° . Work out the height of the mast.

[4 marks]

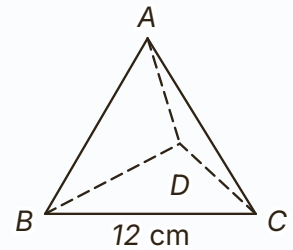


Answer m

Not drawn accurately

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** $ABCD$ is a regular tetrahedron: each of its six edges is 12 cm. Work out the angle between two of its faces. [4 marks]



Answer°

Not drawn accurately

Extension hints

Cover these with paper. Uncover one at a time.

- E1 Hint 1** The faces ABC and DBC meet along BC . Let M be the midpoint of BC : AM and DM are both at right angles to BC , so the angle you want is AMD .
- Hint 2** AM and DM are heights of equilateral triangles of side 12: $AM = DM = \sqrt{12^2 - 6^2} = \sqrt{108}$. And $AD = 12$.
- Hint 3** $\cos AMD = \frac{108 + 108 - 144}{2 \times 108} = \frac{1}{3}$, so the angle is 70.5° (whatever the length of the edges).

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can find lengths in a solid using Pythagoras in three dimensions. ☐ ☐ ☐
- I can find the angle between a line and a plane. ☐ ☐ ☐
- I can find the angle between two planes. ☐ ☐ ☐
- I can use the sine and cosine rules in three-dimensional problems. ☐ ☐ ☐