



Quadratic equations

About 85 minutes Calculator allowed 65 marks

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] **A2** GCSE Higher [2 marks]
- Factorise $6x^2 - 7x - 10$.
- Write $\sqrt{252}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible.

Answer

Answer

- A3** Last lesson [2 marks] **A4** From chapter 2 [2 marks]
- $f(x) = \begin{cases} x + 2, & x < 1 \\ 7 - x^2, & x \geq 1 \end{cases}$
- Work out $f(-3)$ and $f(2)$.
- Write $x^2 - 4x + 9$ in the form $(x + a)^2 + b$.

Answer

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Solve $6x^2 = 7x + 20$.

Hint First move every term to one side, so the other side is 0. Then factorise.

- B1** Solve $10x^2 + 9 = 21x$. [3 marks]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

EXAMPLE 2

Solve $x^2 - 6x - 11 = 0$ by completing the square. Give your answers in exact form.

Hint Halve the coefficient of x and square it. Write the left side as a square, then square root both sides, remembering $\pm$.

- B2** Solve $x^2 + 8x - 2 = 0$ by completing the square. Give your answers in exact form. [3 marks]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

EXAMPLE 3

Solve $3x^2 + 5x - 4 = 0$. Give your answers (a) in exact form, (b) correct to 2 decimal places.

Hint It does not factorise. Write down a , b and c , then use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

B3 Solve $5x^2 - 2x - 4 = 0$.

- (a) Give your answers in the [2 marks]
form $\frac{a \pm \sqrt{b}}{c}$, where a , b
and c are integers and b
is as small as possible.

Answer $x = \dots\dots\dots$

- (b) Give your answers [2 marks]
correct to 2 decimal
places.

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

Total for B3: 4 marks

EXAMPLE 4

Solve $\frac{4}{x-1} - \frac{3}{x} = 1$.

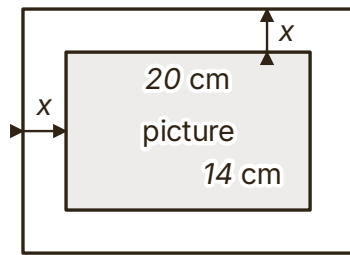
Hint Multiply every term by $x(x-1)$ to clear the fractions. Then collect all terms on one side.

B4 Solve $\frac{5}{x-3} + \frac{6}{x+1} = 2$. [4 marks]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

EXAMPLE 5

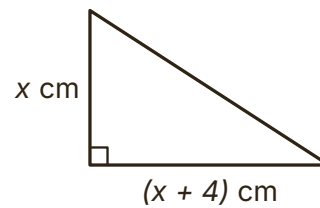
A picture 20 cm wide and 14 cm high is in a frame of the same width x cm all the way round. The total area of the picture and frame is 432 cm^2 . Work out x .



Not drawn accurately

Hint Write the outside width and height in terms of x , then multiply them. A width cannot be negative.

B5 A right-angled triangle has shorter sides of x cm and $(x + 4)$ cm. Its area is 25 cm^2 .



Not drawn accurately

(a) Show that [2 marks]
 $x^2 + 4x - 50 = 0$.

(b) Work out the value of x . [2 marks]
 Give your answer correct to 2 decimal places.

Answer $x = \dots\dots\dots$

Total for B5: 4 marks

c Practice

Level 2

C1 Solve $9x^2 + 30x + 25 = 0$. How many different solutions are there? [2 marks]

Answer

C2 Work out the value of $b^2 - 4ac$ for the equation $3x^2 + 4x + 2 = 0$. Hence write down the number of real roots of the equation. [2 marks]

Answer

- C3** Two positive numbers differ by 3. The sum of their squares is 117. Work out the two numbers. [3 marks]

Answer and

- C4** A stone is thrown up from the top of a wall. After t seconds its height is h metres, where $h = 2 + 19t - 5t^2$. [3 marks]
- (a) Work out the two times when the stone is 14 m high.

Answer $t = \dots\dots$ and $t = \dots\dots$

- (b) Work out the time when the stone reaches the ground. [2 marks]
Give your answer correct to 2 decimal places.

Answer $t = \dots\dots$

Total for C4: 5 marks

- C5** The formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ is used for a lens. For one lens $f = 12$, and v is 10 more than u . Work out u , which is positive. [3 marks]

Answer $u = \dots\dots$

- C6** Sam used the quadratic formula and correctly found $x = \frac{-3 \pm \sqrt{29}}{2}$. Write down a quadratic equation $x^2 + bx + c = 0$ that Sam might have solved. [2 marks]

Answer

D Exam-style questions

AQA exam style

- D1** The equation $x^2 + 6x + c = 0$ has one repeated root. [1 mark]
Circle the value of c .
3 6 9 36

D2

Leo is solving $(x - 2)(x + 5) = 30$. He writes: " $x - 2 = 30$ or $x + 5 = 30$, so $x = 32$ or $x = 25$."

Leo is wrong.

(a) Show that $x = 32$ is not a solution. [1 mark]

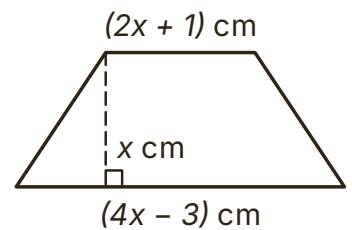
(b) Solve $(x - 2)(x + 5) = 30$ correctly. [3 marks]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

Total for D2: 4 marks

D3 The diagram shows a trapezium. Its area is 24 cm^2 .

(a) Show that $3x^2 - x - 24 = 0$. [2 marks]



(b) Hence work out the length of the longer parallel side. [3 marks]

Not drawn accurately

Answer $\dots\dots\dots$ cm

Total for D3: 5 marks

D4 Solve $\frac{2}{x-1} - \frac{1}{x+2} = \frac{1}{3}$. [5 marks]

Give your answers to 3 significant figures. You must show your working.

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

- D5** The solutions of the equation $x^2 + bx + c = 0$ are $x = 2 + \sqrt{7}$ and $x = 2 - \sqrt{7}$. [3 marks]
Work out the values of b and c .

Answer $b = \dots\dots$, $c = \dots\dots$

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Solve $(x^2 - 3x)^2 - 14(x^2 - 3x) + 40 = 0$. [4 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Do not expand. The same expression appears twice: give it a single letter.
Hint 2 Let $u = x^2 - 3x$. Then $u^2 - 14u + 40 = 0$, so $(u - 4)(u - 10) = 0$.
Hint 3 $x^2 - 3x = 4$ gives $(x - 4)(x + 1) = 0$; $x^2 - 3x = 10$ gives $(x - 5)(x + 2) = 0$. So $x = -2, -1, 4$ or 5 .

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can solve a quadratic equation by factorising, by completing the square or with the formula. ☐ ☐ ☐
- I can give the answers in exact form or to a given accuracy. ☐ ☐ ☐
- I can turn an equation with fractions, or a problem in context, into a quadratic and solve it. ☐ ☐ ☐
- I can tell from $b^2 - 4ac$ how many real roots an equation has. ☐ ☐ ☐