

Sequences and limiting values

About 85 minutes No calculator 73 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/further-maths-sequences-and-limiting-values/

Date _____

A Do Now

The first two questions are what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks]

Solve $2x^2 - 5x - 12 = 0$.

A2 Needed today [2 marks]

Work out the n th term of the linear sequence 17, 11, 5, -1, ...

Answer $x = \dots\dots$ or $x = \dots\dots$

Answer

- A3** Last lesson [2 marks]

Prove that $(n + 5)^2 - (n + 1)^2$ is a multiple of 8 for every integer n .

A4 From chapter 3 [2 marks]

$f(x) = 2x + 1$ and $g(x) = x^2$.
Work out $fg(3)$.

Answer

B Example, then your turn

Level 2

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Work out the n th term of the quadratic sequence 5, 12, 23, 38, 57, ...

Hint The second differences are all 4, so the sequence starts $2n^2$. Take $2n^2$ away from each term and find the n th term of what is left.

- B1** Work out the n th term of the quadratic sequence 1, 9, 23, 43, 69, ... [3 marks]

Answer

EXAMPLE 2

The 1st, 3rd and 5th terms of a quadratic sequence are 6, 26 and 62. Work out the n th term of the sequence.

Hint Write the n th term as $an^2 + bn + c$. Put $n = 1, 3$ and 5 in to get three equations, then eliminate c .

B2 The 1st, 2nd and 4th terms of a quadratic sequence are 2, 9 and 35. Work out the n th term of the sequence.

Answer

EXAMPLE 3

The n th term of a sequence is $\frac{6n+5}{4n-1}$. Prove that the limiting value of the sequence as $n \rightarrow \infty$ is $\frac{3}{2}$.

Hint Divide the top and the bottom by n . Then think about what happens to $\frac{5}{n}$ and $\frac{1}{n}$ as n gets very large.

B3 The n th term of a sequence is $\frac{9-2n}{5n+3}$. Prove that the limiting value of the sequence as $n \rightarrow \infty$ is -0.4 .

EXAMPLE 4

The n th term of a sequence is $\frac{3n+1}{2n+3}$. Which term has the value 1.4? Show that 1.6 is not a term of the sequence.

Hint Set the n th term equal to the value and solve for n . A term's position must be a positive whole number.

B4 The n th term of a sequence is $\frac{5n-2}{3n+1}$. Work out which term has the value 1.6, and show that 2 is not a term of the sequence.

Answer the th term

EXAMPLE 5

The n th term of a sequence is $n^2 + 3n$. Which two consecutive terms have a sum of 158?

Hint The next term after the n th is $(n + 1)^2 + 3(n + 1)$. Add the two terms, set the sum equal to 158 and solve.

B5 The n th term of a sequence is $2n^2 - n$. Which two consecutive terms have a sum of 211?

Answer the th and th terms

c Practice

Level 2

C1 A linear sequence starts 4, 7.5, 11, 14.5, ... Work out the value of the first term that is greater than 300. [3 marks]

Answer

C2 A linear sequence is $a, a + 3b, a + 6b, a + 9b, \dots$. The 2nd term is 23 and the 5th term is 59. Work out the values of a and b , and the n th term of the sequence. [3 marks]

Answer $a = \dots$ $b = \dots$ n th term =

C3 Work out the n th term of the linear sequence 3, 7, 11, 15, ... Hence work out the n th term of the sequence 9, 49, 121, 225, ... in the form $an^2 + bn + c$. [3 marks]

Answer

C4 The n th term of a sequence is $n^2 - 16n + 67$. Prove that every term of the sequence is positive. [3 marks]

C5 A sequence starts with 3. Each term after the first is found by multiplying the previous term by 3 and then subtracting 4. Work out the 4th term. Explain why no term of the sequence is even. [3 marks]

Answer 4th term =

C6 Work out the limiting value, as $n \rightarrow \infty$, of the sequence with n th term $\frac{6n^2 - 5n}{3n^2 + 2n + 7}$. [2 marks]

Answer

D1

Mia says, "The sequence with n th term $\frac{5n+2}{n+1}$ never gets close to 5, so 4.9 is not one of its terms."

n is a positive integer.

- (a) A term of the sequence has a value of 4.7. Work out the value of n . [2 marks]

Answer $n = \dots\dots$

- (b) Write down the limiting value of the sequence as $n \rightarrow \infty$. [1 mark]

Answer

- (c) Show that Mia is wrong. [2 marks]

Total for D1: 5 marks

D2

For sequence A, n th term $= \frac{11n+40}{n}$. For sequence B, n th term $= 2n$.

- (a) The k th term of sequence A equals the k th term of sequence B. Work out the value of k . You must show your working. [4 marks]

Answer $k = \dots\dots$

- (b) Write down the limiting value of sequence A as $n \rightarrow \infty$. [1 mark]

Answer

Total for D2: 5 marks

D3 The first five terms of a quadratic sequence are 4, 3, 0, -5, -12

- (a) Work out an expression for the n th term. [3 marks]

Answer

(b) Work out which term of the sequence is equal to -192 .

[2 marks]

Answer the th term

Total for D3: 5 marks

D4 The n th term of a sequence is $\frac{an+6}{bn-1}$, where a and b are positive integers. The 1st term of the sequence is 6. The limiting value of the sequence as $n \rightarrow \infty$ is 3. Work out the value of the 2nd term of the sequence. [4 marks]

Answer

D5 The n th term of a sequence is $T_n = \frac{2n+1}{n+2}$.

(a) Show that $T_{n+1} - T_n = \frac{3}{(n+2)(n+3)}$. [3 marks]

(b) Hence explain why each term of the sequence is greater than the term before it. [1 mark]

(c) Work out the first value of n for which $T_{n+1} - T_n < 0.01$. [2 marks]

Answer $n = \dots\dots$

(d) Write down the limiting value of T_n as $n \rightarrow \infty$. [1 mark]

Answer

Total for D5: 7 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The n th term of a sequence is $\frac{3n+7}{n+1}$. Find every term of the sequence that is a whole number, and prove that there are no others. [4 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write the n th term as 3 plus a fraction.
- Hint 2** $3n+7 = 3(n+1) + 4$, so the n th term is $3 + \frac{4}{n+1}$.
- Hint 3** It is a whole number only when $n+1$ divides 4. With $n \geq 1$, $n+1 = 2$ or 4, so $n = 1$ (term 5) or $n = 3$ (term 4). For $n \geq 4$, $0 < \frac{4}{n+1} < 1$, so no other term is whole.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can work out the n th term of a linear or a quadratic sequence. ☐ ☐ ☐
- I can use the n th term to find a term's position, or show a value is not a term. ☐ ☐ ☐
- I can find and prove the limiting value of a sequence as $n \rightarrow \infty$. ☐ ☐ ☐
- I can solve problems that combine two terms or two sequences. ☐ ☐ ☐